

For problem VII A on Midterm review sheet:

> with(Groebner) :

> BA := Basis([x² - 2·y·z, x - y² - 2, z - x·y], plex(x, y, z));
BA := [z³ - 32 z, 4 yz - z², 8 y² - z² + 16, -z² + 8 x] (1)

> NormalForm(x³ + y³ + z³ - 2 z² + 2·y - 33·z, BA, plex(x, y, z));
0 (2)

For problems VII B and C on Midterm review sheet:

> BB := Basis([x² + y³ + z³ - 1, x - y² + z - 1, x² - y² - z + 1], plex(x, y, z));
BB := [z¹⁰ + 2 z⁹ + 11 z⁸ + 8 z⁷ + 20 z⁶ - 25 z⁵ - 3 z⁴ - 58 z³ + 52 z² - 40 z + 32, (3)
- 83 z⁹ - 128 z⁸ - 809 z⁷ - 566 z⁶ - 1416 z⁵ + 635 z⁴ + 1411 z³ + 2512 yz
+ 2912 z² - 2512 y + 540 z - 2496, y³ + z³ + y² + z - 2, -y² + x + z - 1]

> for i to nops(BB) do BB[i] end do;
z¹⁰ + 2 z⁹ + 11 z⁸ + 8 z⁷ + 20 z⁶ - 25 z⁵ - 3 z⁴ - 58 z³ + 52 z² - 40 z + 32
- 83 z⁹ - 128 z⁸ - 809 z⁷ - 566 z⁶ - 1416 z⁵ + 635 z⁴ + 1411 z³ + 2512 yz
+ 2912 z² - 2512 y + 540 z - 2496
y³ + z³ + y² + z - 2
- y² + x + z - 1 (4)

> factor(BB[1]);
(z - 1) (z⁹ + 3 z⁸ + 14 z⁷ + 22 z⁶ + 42 z⁵ + 17 z⁴ + 14 z³ - 44 z² + 8 z - 32) (5)

> collect(BB[2], y);
(2512 z - 2512) y - 83 z⁹ - 128 z⁸ - 809 z⁷ - 566 z⁶ - 1416 z⁵ + 635 z⁴ (6)
+ 1411 z³ + 2912 z² + 540 z - 2496

> factor(BB[2]);
(z - 1) (-83 z⁸ - 211 z⁷ - 1020 z⁶ - 1586 z⁵ - 3002 z⁴ - 2367 z³ - 956 z² (7)
+ 2512 y + 1956 z + 2496)

> BB[3];
y³ + z³ + y² + z - 2 (8)

> BB[4];
-y² + x + z - 1 (9)

> fsolve(BB[1], z, complex);
-1.11194873768170 - 1.18550014787317I, -1.11194873768170 (10)
+ 1.18550014787317I, -0.501775886607404 - 1.78835410228050I,
-0.501775886607404 + 1.78835410228050I, -0.441018760972384

– 2.52690155393694I, –0.441018760972384 + 2.52690155393694I,
0.115124189048988 – 0.770478927916720I, 0.115124189048988
+ 0.770478927916720I, 0.879238392425002, 1.000000000000000

