MATH 400 – Directed Readings in Complex Functions and Riemann Surfaces
Spring 2014

Course Format

The course will be closely based on topics from the book *Complex Functions, an algebraic and geometric viewpoint*, by G. Jones and D. Singerman (I have a copy for you).

We will meet for about 2 hours each Monday afternoon during the semester in my office, Swords 331. The assignment for each meeting will include some reading from the Jones and Singerman book (to be done *before* our meeting), plus some problems (starting with the second meeting). I'm hoping to spend about a half hour each week after the first on questions that might have come up from the reading, and having you present some parts of solutions to the problems orally. Then I'll do a "lecture" on some new material. But we will not prove every theorem in the book in detail in these lectures, as you might expect in a more formal course; some of your work each week should be figuring out some of these arguments individually.

For formal assessments, I will give a larger midterm problem set around the middle of the semester, and then either a similar final problem set or a project paper on a topic related to the mathematics we study (we will discuss this, and I will let you choose which option fits your goals and interests better).

(Very) Tentative Schedule

Note: This might be too ambitious, but it would be very nice to cover this much of the book. There's no real pressure, though, and we won't rush to fit this much in if it turns out we need to go more slowly. We'll see that even covering the material in Chapters 1 - 3 will give a nice follow-up to the Complex Analysis course and tie things you have seen in other courses together in a beautiful way as a "capstone" course.

- Week 1 – January 27: The Riemann sphere Σ and mermorphic functions. Reading: Appendix 1 and §§1.1 - 1.4; no problems due this first meeting.
- Week 2 – February 3: Meromorphic functions as mappings $f : \Sigma \to \Sigma$, Möbius transformations. Reading: §1.5, §§2.1 - 2.9. Problems: to be assigned.
- Week 3 – February 10: Geometric classification of Möbius transformations, finite subgroups of the group of Möbius transformations. Reading: §§2.10 - 2.15. Problems: to be assigned.
- Week 4 – February 17: Discrete subgroups of $\mathbf{C}, +$, periodic functions. Reading: §§3.1 - 3.6. Problems: to be assigned.
- Week 5 – February 24: Sum and product expansions, the Weierstrass $\zeta, \sigma, \wp$ functions. Reading: §§3.7 - 3.9. Problems: to be assigned.
- Spring Break – week of March 3
- Week 6 – March 10: The differential equation of the $\wp$-function, the field of elliptic functions. Reading: §§3.10 - 3.12. Problems: to be assigned.
- Week 7 – March 17: Elliptic functions with given zeroes and poles; with given principal parts. Reading: §§3.13 - 3.15. Problems: Midterm problem set.

- Week 8 – March 24: Elliptic curves, the group law, applications. Reading: §§3.16-3.17. Problems: to be assigned.
- Week 9 – March 31: Analytic/meromorphic continuation. Reading §§4.1 - 4.4. Problems: to be assigned.
- Week 10 – April 7: Homotopy of curves; the monodromy theorem. Reading §§4.5 - 4.6. Problems: to be assigned.
- Week 11 – April 14: Riemann surfaces of $\log z, z^{1/q}, \sqrt{p(z)}$. Reading: §§4.7 - 4.9. Problems: to be assigned.
- Easter break includes Monday, April 21
- Week 12 – April 28: Abstract Riemann surfaces. Reading: §§4.10 - 4.11. Problems: to be assigned.
- Week 13 – May 5: Function theory on Riemann surfaces, course wrap-up. Reading: §§4.12 - 4.16. Problems: Final problem set, or project.