Mathematics 375 – Probability and Statistics 1
Solutions For Whole Class Discussion Problems – The "Event-Composition Method"
September 16, 2011

Discussion Questions

A) Given: $P(A) = .2$, $P(B) = .3$, $P(A \cup B) = .4$. Determine:

1) $P(\overline{A} \cup \overline{B})$.
Solution: By the additive law, we have

$$P(\overline{A} \cup \overline{B}) = P(\overline{A}) + P(\overline{B}) - P(\overline{A} \cap \overline{B}).$$

Since $P(A) = .2$, $P(\overline{A}) = 1 - .2 = .8$. Similarly, $P(\overline{B}) = 1 - .3 = .7$. Finally, $\overline{A} \cap \overline{B} = \overline{A \cup B}$ by one of the DeMorgan Laws from set theory. So $P(\overline{A} \cap \overline{B}) = 1 - .4 = .6$. Putting everything together,

$$P(\overline{A} \cup \overline{B}) = .8 + .7 - .6 = .9$$

2) $P(\overline{A}|B)$.
Solution: By the definition of conditional probability,

$$P(\overline{A}|B) = \frac{P(\overline{A} \cap B)}{P(B)}$$

We know $P(B) = .3$ from the given information. In the course of solving part A, we saw $P(\overline{A} \cap \overline{B}) = .6$ and $P(\overline{A}) = .8$. Since

$$\overline{A} = (\overline{A} \cap B) \cup (\overline{A} \cap \overline{B}),$$

and the two intersections in the parentheses don't intersect each other, we have

$$.8 = P(\overline{A} \cap B) + .6, \text{ hence } P(\overline{A} \cap B) = .2$$

Then

$$P(\overline{A}|B) = \frac{.2}{.3} \doteq .67.$$

B) The circles marked 1,2,3,4 in the diagrams on the sheet given in class represent electrical relays that operate independently and function properly with probability $p = .85$. Which design yields the higher probability that current will flow when the relays are activated?

Solution: Let $1, 2, 3, 4$ represent the events that the corresponding relays function correctly. Layout I allows the current to flow if either 1 and 2 function correctly, or 3 and

4 do, or both sets of two do. So the probability we want is $P((1 \cap 2) \cup (3 \cap 4))$. By the additive law and independence, this is

$$\begin{aligned} P(1 \cap 2) + P(3 \cap 4) - P(1 \cap 2 \cap 3 \cap 4) &= (.85)^2 + (.85)^2 - (.85)^4 \\ &\doteq .923. \end{aligned}$$

Similarly, Layout II allows the current to flow if any one of the 4 relays functions correctly. The current *does not* flow only if all four relays fail, so we can compute the probability we want as

$$1 - P(\overline{1} \cap \overline{2} \cap \overline{3} \cap \overline{4}) = 1 - (.15)^4 \doteq .9995.$$

As you should have seen intuitively, Layout II gives a significantly higher probability.

C) An accident victim will die unless he receives type $A+$ blood (strictly) before 8 minutes elapse. He will be saved if he does get the blood transfusion. Potential donors and a reusable blood typing kit are available, but it takes 2 minutes to determine each possible donor's blood type, and only 40% of them have type $A+$ blood. What is the probability that the victim will be saved if only one donor's blood can be typed at a time? (Follow-up: Are you making an assumption in your solution?)

Solution: There are just three chances for an $A+$ donor to be found since each typing takes 2 minutes and the blood has to be given to the victim strictly before 8 minutes have passed. The chance of finding a good donor on the first test is .4. When that test fails, there is a (.6)(.4) chance of success on the second test. Similarly, when the first two tests fail, there is a $(.6)^2(.4)$ chance of success on the last test before time runs out. The total probability of finding a donor and saving the victim is

$$(.4) + (.4)(.6) + (.4)(.6)^2 = .784$$

More formally, say F is the event that a match for the blood is found on or before the third try. Then

$$F = F_1 \cup F_2 \cup F_3$$

where F_i = *first match found on try* i, for $i = 1, 2, 3$. Then $F_i \cap F_j = \emptyset$ if $i \neq j$, so

$$P(F) = P(F_1) + P(F_2) + P(F_3).$$

As before $P(F_1) = .4$. But then F_2 = match found on the second person but not on the first. So $P(F_2)$ is the probability that the first person tested does not have type $A+$ blood, and the second person does have type $A+$ blood. The value (.4)(.6) is OK here (only) as long as the two people's blood types are *independent*. Similarly for the $(.4)(.6)^2$ in the last term.