Mathematics 375 – Probability Theory
Information on Exam 1
September 28, 2011

General Information

As announced in the course syllabus, the first midterm exam will be given next week during our regular class time on Friday October 7. The exam will cover the material we have discussed since the beginning of the semester, up to and including the material on binomial, geometric, and hypergeometric discrete random variables. (This is the same as the material from Problem Sets 1 - 4 and the material from class on Friday, September 30). In more detail, the topics to be covered are:

1) Descriptive statistics such as the mean, standard deviation, and frequency histograms for numerical data, the "empirical rule"
2) Discrete sample spaces and counting techniques for sample points – the $m \cdot n$ rule, permutations, binomial and multinomial coefficients
3) The "Sample Point Method" for probabilities
4) Conditional probability and independence of events
5) The Law of Total Probability and Bayes' Rule – *know proofs of these* and how to apply them.
6) The "Event Composition Method" for probabilities
7) Discrete random variables, probability functions, expected value, variance – *know the proof of the equation* $V(Y) = E(Y^2) - (E(Y))^2$ *as well as how to apply it.*
8) Binomial random variables – *know the proof of the formula* $E(Y) = np$ *for a binomial random variable based on* n *trials with success probability* p, and the formula for variance of binomial random variables.

Other Information and Groundrules

You may prepare *one side of a* 3×5 *inch index card* with formulas, bring it to the exam and consult it at any time. You may *not* put the proofs of any of the results indicated above on your formula card. The cards will be collected with the exams and returned with the graded exams. No electronic devices other than calculators will be allowed.

Review Session

I will be happy to run a pre-exam review session, however I have evening commitments on Monday, Tuesday, and Wednesday next week.

Suggested Review Problems

From the text: 1.22, 1.24, 1.31, 2.143, 2.144, 2.145, 2.146, 2.147 (Note: the "full house" would have to consist of either the pair of aces and three kings, or a pair of kings and three aces. You should assume that a single deck of cards is being used, and that *only* the original five cards have been dealt before the discard and the deal of the two additional cards – there are no other players.), 2.148 (note: this is sampling *without* replacement; what would happen if the components were replaced in the bin each time before the next one was drawn?), 2.149, 2.150, 2.153, 2.155, 2.163, 3.15, 3.32, 3.53, 3.56, 3.61.

Sample Exam Questions

These come from first midterm exams in previous offerings of MATH 375.

I. A manufacturer of electronic components tests the lifetimes of a certain type of battery and finds the following data:

$$123, 116, 122, 110, 125, 126, 111, 118, 117, 120$$

(lifetimes in hours). How many of the sample points are within one standard deviation of the sample mean? Is there reason to believe the lifetime of this type of battery is not normally distributed from this small sample? Explain.

II. In a regional spelling bee, the 10 finalists consist of 5 girls and 5 boys. Assume that all the finalists are equally proficient spellers and that the outcome of the contest is random. What is the probability that 4 of the top 5 finishers will be female?

III.
- A) State and prove the Law of Total Probability.
- B) The Podunk City police department plans a crackdown on speeders by placing radar traps at four different locations L_1, L_2, L_3, L_4. The probability that each of the traps is manned at any one time is .4, .3, .2, .3 respectively. The police really mean business – if a trap is manned every speeder who passes it will get a ticket. Speeders have probabilities of passing the four locations of .2, .1, .5, .2 respectively, and no one passes more than one. What is the probability that a given speeder will actually receive a ticket?
- C) In the situation of part B, given that a speeder received a ticket, what is the probability that he passed location L_2?

IV. Let A, B be events for which $P(A) = .2$, $P(B) = .3$ and $P(A \cap B) = .06$. Are $\overline{A}$ and $\overline{B}$ independent events? Explain your assertion.

V. An allergist knows that 30% of all people are allergic to the pollen of burdock weed. The allergist sees 20 patients in all on one day.
- A) What is the probability function for the random variable Y = number of patients among the 20 who are allergic to burdock pollen? Explain the assumptions you are making to derive your solution.
- B) What are the expected value and variance for the Y from part A?
- C) What is the probability that from 4 to 7 (inclusive) of the 20 patients she sees are allergic to burdock pollen?

D) What is the probability that the first patient the doctor sees who is allergic to burdock pollen will be the last patient seen? Explain the assumptions you are making to derive your solution.

Extra Credit-Type Question

Let Y have a geometric distribution with success probability p. Show that the expected value of $g(Y) = e^Y$ is $E(e^Y) = \frac{pe}{1-qe}$.