Mathematics 375 – Probability and Statistics 1
Discussion 4 – The "Method of Moment-Generating Functions"
November 30, 2009

Background

Because of the Uniqueness Theorem, if we can compute the moment-generating function for a random variable and recognize it as one of our standard forms, then we know its distribution: that is, its probability density function, mean, variance, and hence "everything about it"(!) Today, we want to use this idea to work several examples and identify what we have.

Discussion Questions

A) First, we will verify one point that we deferred in discussing the use of the standard normal table. Recall, we said that if Y is normal with mean μ and standard deviation σ, then

$$Z = \frac{Y - \mu}{\sigma}$$

would have a standard normal distribution (i.e. normal with mean 0 and standard deviation 1). We never really justified this claim before in class. (It did come up on one of the review problems for Exam 2.) But we *can do it now*! Note that

$$Z = \frac{1}{\sigma}Y - \frac{\mu}{\sigma}.$$

Find the moment generating function of Z given the moment generating function for the normal Y:

$$m_Y(t) = e^{\frac{t^2\sigma^2}{2} + \mu t}.$$

Recall our formula $m_{aY+b}(t) = e^{bt}m_Y(at)$. Deduce that Z must have a standard normal distribution.

B) Let Y_1 and Y_2 be independent standard normals, and let $U = Y_1^2 + Y_2^2$.

1) Set up the integral to compute

$$m_U(t) = E(e^{tU})$$

using the joint density for Y_1, Y_2.

2) For the rest of the problem, assume $1 - 2t > 0$. Combine terms in your integral, make the substitutions $u_i = \sqrt{1 - 2t} \cdot y_i$, and show that

$$m_U(t) = \frac{1}{\sqrt{1 - 2t}} \cdot \frac{1}{\sqrt{1 - 2t}} = \frac{1}{1 - 2t}.$$

3) What is the distribution of X? (What "type of random variable" is U, according to the Uniqueness Theorem?)

4) Suppose $Y_1, \dots, Y_k$ are independent random variables, each with a standard normal distribution. What is the distribution of $U = Y_1^2 + \cdots + Y_k^2$?

Assignment

Group writeups due in class on Monday, December 7.