Mathematics 376 – Probability and Statistics II
Lab Project 3 – Hypothesis Testing
April 6, 2010

Background

We have now discussed various tests of hypotheses for means of normal distributions and proportions. In this lab we will work through several realistic examples to illustrate the thinking process statisticians would use to select appropriate tests, and interpret the results.

New Maple Command

There is another graphical routine in our statistics package that produces what are called "box-and-whisker" plots for lists of numerical data. You will need to use these at several points in this lab. The idea is the following. To get a rough visual picture of the distribution of a data set, as an alternative to the relative frequency histogram, we can use a *box-and-whisker* plot. The idea of this graphical display is to show the locations of the *minimum*, the *25*th percentile, the median or *50*th percentile, the *75*th percentile and the maximum of the data values by drawing a box with vertical bars at the 25th, 50th, 75th percentile values, together with two thinner "whiskers" extending out to the minimum and maximum. The new procedure in the Maple package is designed to draw one of these plots for any collection of lists, "stacked vertically," in one graphical display so that we can visually compare data sets in a rough way. Check the on-line documentation for more information and a usage example.

Lab Questions

A) An office furniture manufacturer has developed a new glue application process for assembling tables. To compare the new process with the old one currently in use, random samples with $n = 30$ are selected from inventories produced with the two processes. Each table is subjected to "destructive testing" in which the force (in pounds) needed to break the glue in the table was measured. Let X be the force in pounds needed to break one of the new tables, and Y be the force in pounds needed to break one of the old tables. The goal is to determine whether the new gluing process has significantly increased the strength of the tables. The data collected was as follows:

	1250	1210	990	1310	1320	1200	1290	1360	1200	1150
X :	1120	1360	1310	1110	1320	980	950	1430	1100	1080
	960	1050	1310	1240	1420	1170	1470	1060	1230	1300

and

	1180	1360	1310	1190	920	1060	1440	1010	1000	950
Y :	1310	980	1310	1030	960	800	1280	1080	900	1030
	930	1050	1010	1310	940	860	1450	1070	840	1100

1) Construct box-and-whisker plots for these data sets (plotted together) and make an informal conjecture about whether or not the new process (the X data) has increased the strength of the tables, compared with the Y data.
2) Describe an appropriate test of the null hypothesis $H_0 : \mu_X = \mu_Y$ versus $H_a : \mu_X > \mu_Y$. Say what your assumptions about the data are, what your test statistic is, what the rejection region will be, and so forth.
3) Carry out your test at the $\alpha = .01$ level of significance. Give a clear and concise statement of the conclusion you draw from your test.
4) What is the attained significance level of your test (the p-value)? Use the appropriate CDF function in the Maple package to determine an accurate estimate of p, not just a range of possible values. What does this say?

B) Let X be the lengths of male spiders of a particular large species and let Y be the lengths of female spiders of the same species (both in mm). Assume that the distributions of X, Y are normal: $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$ respectively. A random sample of $n_X = 9$ X values were taken:

$$20.4, 21.7, 21.9, 21.4, 21.1, 23.6, 18.9, 22.6, 21.3$$

Similarly, a random sample of $n_Y = 13$ observations of Y were made:

$$20.5, 20.4, 20.3, 21.1, 21.2, 20.9, 21.0.21.3, 20.9, 20.0, 20.4, 20.8, 20.3$$

Is there a statistically demonstrable difference in the length distributions of two sexes, though? We know the t-test that applies in that case, but recall that that test is based on the assumption that the two sets of measurements come from normal populations, *with equal variances.* Here is one possible procedure for testing for equality of two normal distributions in the small sample case:

- First, test for equality of variances. This will be based on an F-statistic as described in section 5.9 of the text (pages 530-537). We have not discussed this explicitly in class, but the idea is very similar to what we did before for confidence intervals for the ratio of two variances. So you should be able to look up what you need and follow the book's discussion.
- If there is *no demonstrable difference* in the variances, test for equality of the means using the t-test for equality of means that we discussed in class (using the pooled estimator S_p for the common variance).
- If there *is a demonstrable difference* in the variances, the basic t-test is not all that reliable in some cases. With small sample sizes, many experienced statisticians would use a different approximating distribution due to Welch: Use the test statistic:

$$t = \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}}},$$

but set up the rejection region for the test using a t-distribution with r degrees of freedom where

$$r = \left[\frac{(S_X^2/n_X + S_Y^2/n_Y)^2}{(S_X^2/n_X)^2/(n_X-1) + (S_Y^2/n_Y)^2/(n_Y-1)} \right] \tag{1}$$

(where $[z]$ = greatest integer less than or equal to z).

In this problem, you will follow this procedure to test this data:

1) Test the null hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$ against the alternative hypothesis $H_a : \sigma_X^2 \neq \sigma_Y^2$. You may select the significance level α. Interpret your results and give the attained significance level (p-value).
2) Test the null hypothesis $H_0 : \mu_X = \mu_Y$ against the alternative hypothesis $H_a : \mu_X^2 \neq \mu_Y^2$. Select the test statistic, etc. based on the results of your test from part 1 and explain your choice. Also clearly state the conclusion you draw from the test.
3) Construct box-and-whisker plots of the two data sets and reconcile with your results in parts 1 and 2.

Assignment

Group write-ups due *at the end of the class on Monday, April 12.*