Mathematics 375 – Probability and Statistics 1
Problem Set 3
due: In class, Thursday September 24

From the text: 2.71, 72, 74, 75, 81, 84, 85, 88, 95, 97.

Additional Problem

Much of the mathematical theory of probability was first developed (ironically enough) to analyze what happens in *games of chance* including various card and dice games used for gambling. One famous example is the dice game "craps" (a mainstay of mobster movies, *The Sopranos*, etc.) In case you have never played, the rules of craps are the following:

1) At the start of the game, the player rolls two dice.
2) If the first total is 7 or 11, then the player wins immediately.
3) If the first total is 2,3, or 12, then the player loses immediately.
4) If the first total is anything else, that becomes the player's "point". The player rolls the dice again (repeatedly) until either the point total is obtained again, in which case the player wins, or else a 7 is rolled, in which case the player loses. *Any number of additional rolls is possible in this case; the game continues until either the point is rolled again, or a 7 is rolled.*

There is a Maple package for probability calculations containing a craps simulator procedure on our course homepage. That simulator can be used to generate any number of craps games and study the outcomes. After you have loaded the package (see directions in the accompanying documentation file), you call the procedure with a command of the form $Craps(n, verbose)$; where n is the number of games you want to simulate, and *verbose* (true or false) tells Maple how much output to print out as the games are played.

- *Craps(10,true);* – plays 10 full games and prints all rolls as it goes along. The final output is a list of 10 0's and 1's (0 = player loses in that game; 1 = player wins).
- *Craps(100,false);* – plays 100 full games and generates the final output, but doesn't show all the individual rolls.

1) Using either the *Craps* procedure or actual dice, play 5 games, print out all the rolls, and explain the outcome of each of the games.
2) Experiment with larger numbers of games. What can you say about the apparent *probability* that the player wins an individual game? (Note: you'll probably want to do lots – 100 at least. This is what is nice about the Maple procedure; you can do lots of games fast. But of course, you can also do this "manually" with physical dice if you want.)
3) Now, we'll analyze the game using the probability concepts we have learned. We'll assume the dice used are "fair" (i.e. the probability of each possible number on each die is 1/6).
   a) What is the probability of winning immediately (on the first roll)? What is the probability of losing immediately?

b) Now the tricky part. What is the probability of winning if your point was a 4 or a 10? a 5 or a 9? a 6 or an 8? (Recall, the game can go on for any number of rolls!)

c) What is the total probability of winning? How well does this match your simulation results?

Note: Casino games are not likely to be popular if the player wins too rarely. Your answer for part 3 should give some evidence why “craps” is a popular game, but one that casino operators can make money on too(!)