Mathematics 375 – Probability and Statistics 1
Information on Exam 2
November 10, 2009

General Information

The second midterm exam will be given Thursday evening November 19. The exam will cover the material we have discussed since the first exam, up to and including the material on joint densities from Section 5.2. (In other words, this is the material from Problem Sets 5 - 8.)

Format and Groundrules

This will be a *closed book* exam, but you may prepare *one side of a* 3×5 *inch index card* with formulas and any other information you want to include, bring it to the exam and consult it at any time. I will provide copies of the tables for Poisson and normal probabilities from the text. Calculators allowed. No cell phones, I-pods, pagers or other electronic devices allowed.

Topics

The topics to be covered are:

1) The hypergeometric and Poisson discrete probability distributions – Sections 3.7 and 3.8.
2) Moment-generating functions for discrete random variables – Section 3.9.
3) Continuous random variables and probability distributions, cdfs and pdfs – Section 4.2
4) Expected values for continuous random variables – Section 4.3
5) Uniform, Normal, Gamma, and Beta distributions – Note that this includes the definition and properties of the Gamma and Beta functions. Be prepared for problems like those on the homework where you need to find the mean and variance of some *function of* a random variable of one of these types, and also for problems where you might use the geometric or binomial distributions in combination with one of these – Sections 4.4 - 4.7
6) Moment-generating functions for continuous random variables – Section 4.9
7) Tchebysheff's Theorem – Section 4.10
8) Joint probability functions, jointly continuous random variables, joint densities – Section 5.2

Review Session

We will review for the exam in class on Tuesday, November 17.

Suggested Review Problems

From the text: Chapter 3/195, 196, 213, Chapter 4/160, 161, 164, 165, 167, 176, 181, 185, Chapter 5/11, 15

Sample Exam Questions

Disclaimer: The following problems represent the approximate level of difficulty of the problems on the upcoming exam. The exam will be about one half of the total length of these problems. The actual exam problems may differ substantially from these.

Comments: Any general formula we have studied for a mean, a variance, etc. can be used without comment (i.e. you don't need to rederive it in you solution).

I. Ten college students, exactly four of whom are not of legal age, go out to a bar and all order alcoholic beverages. The waitperson taking their order selects 5 students at random from the group to "card" and must refuse to serve any one who is under legal age. What is the probability that exactly 2 students will be refused service? (Assume, unrealistically perhaps, that each ID shows that student's actual age!)

II. The fraction Y of impurities in a tank-car lot of industrial grade potassium perchlorate is a random variable with probability density function

$$f(y) = \begin{cases} cy^2(1-y)^2 & \text{if } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

A) What is the value of the constant c?
B) What is the variance of Y?
C) Find the cumulative distribution function for Y.
D) Find $P(.1 \le Y \le .25)$.

III. The life in years of a certain type of electrical switch has an exponential distribution with mean 2 years.

A) What is the probability that a single switch will fail during the first year after it is installed?
B) If 100 of these switches are installed in different, independently operating systems, what is the probability that at most 30 fail during the first year? (You need *not* obtain a single decimal approximation to this number; a formula for computing it will suffice.)

IV. The weights of a population of miniature poodles are normally distributed with mean $\mu = 8$kg and standard deviation $\sigma = .9$kg.

A) What is the probability that a randomly selected poodle will have weight between 7.3kg and 9.1kg?

B) Let Y be the weight of a poodle from the population as a random variable. What is $E(Y^3)$? (Note: part credit will be given for a correct verbal description of a procedure that could be used to derive this value even if you cannot carry out the procedure completely.)

V. Let Y be a random variable with probability density function

$$f(y) = \begin{cases} ye^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the moment-generating function $m_Y(t)$, assuming $t < 1$.

VI. In a shipment of 10 missiles, there are 3 defectives whose engines will not fire. Out of the shipment, 4 missiles are selected at random.

A) What is the probability that all 4 will fire?

B) What is the probability that at most 2 will not fire?

VII. In a biomedical research experiment, the survival time Y in weeks of an animal exposed to a certain fatal disease is a random variable with a gamma distribution with $\alpha = 2$ and $\beta = 6$.

A) What is the mean survival time of a randomly selected exposed animal?

B) The cost of treating an exposed animal is $C = 10 + .1Y^2$. Find the mean and variance of C.

C) If 6 exposed animals are chosen at random, what is the probability that at least two survive for 15 or more weeks?

VIII. The lifetime of a certain type of small motor is normally distributed with mean 10 years and standard deviation 2 years. The manufacturer replaces free all motors that fail while under warrantee. But to avoid going out of business because of the cost of doing the free replacements, the manufacturer must set the warrantee so that no more than 3% of the motors that fail will be covered. What should the length of the warrantee be?

IX.

A) Let Y be a Poisson random variable with mean λ. Find the moment-generating function of $X = aY + b$, where a, b are constant.

B) Let Z be a standard normal random variable. What is the moment-generating function for $Y = Z^2$? What type of random variable is Y? (Hint: You will want to assume that $1 - 2t > 0$ and use the substitution $u = \sqrt{1 - 2t} \cdot y$ to evaluate the integral for $E(e^{tY})$.)