Mathematics 375 – Probability and Statistics 1
Information on Exam 1
October 1, 2009

General Information

As announced in the course syllabus, the first midterm exam will be given next week, either in class or in the evening on Thursday, October 8. The exam will cover the material we have discussed since the beginning of the semester, up to and including the material on binomial and geometric discrete random variables from class on September 29. (This is the same as the material from Problem Sets 1 - 4 and Discussions 1 - 2). In more detail, the topics to be covered are:

1) Descriptive statistics such as the mean, standard deviation, and frequency histograms for numerical data, the "empirical rule"
2) Discrete sample spaces and counting techniques for sample points – the $m \cdot n$ rule, permutations, binomial and multinomial coefficients
3) The "Sample Point Method" for probabilities
4) Conditional probability and independence of events
5) The Law of Total Probability and Bayes' Rule – *know proofs of these* and how to apply them.
6) The "Event Composition Method" for probabilities
7) Discrete random variables, probability mass functions, expected value, variance – *know the proof of the equation* $V(Y) = E(Y^2) - (E(Y))^2$ *as well as how to apply it.*
8) Binomial random variables – *know the proof of the formula* $E(Y) = np$ *for a binomial random variable based on* n *trials with success probability* p, and the formula for variance of binomial random variables.

Other Groundrules

You may prepare *one side of a* 3×5 *inch index card* with formulas and any other information you want to include, bring it to the exam and consult it at any time. Calculators allowed.

Review Session

I will be happy to run a pre-exam review session. If we do the exam in the evening, we could take either Tuesday or Thursday's class meeting for an optional review. We would have regular class meetings the other two days next week, though.

Suggested Review Problems

From the text: 1.22, 1.24, 1.31, 2.143, 2.144, 2.145, 2.146, 2.147 (Note: the "full house" would have to consist of either the pair of aces and three kings, or a pair of kings and three aces. You should assume that *only* the original five cards have been dealt before the discard and the deal of the two additional cards.), 2.148, 2.149, 2.150, 2.153, 2.155, 2.163, 3.15, 3.33, 3.53, 3.56, 3.61, 3.180

Sample Exam Questions

The first five come from the first midterm exam in the Fall 2003 offering of MATH 375.

I. A manufacturer of electronic components tests the lifetimes of a certain type of battery and finds the following data:

$$123, 116, 122, 110, 125, 126, 111, 118, 117, 120$$

(lifetimes in hours). How many of the sample points are within one standard deviation of the sample mean? Is there reason to believe the lifetime of this type of battery is not normally distributed from this small sample? Explain.

II. In a regional spelling bee, the 10 finalists consist of 5 girls and 5 boys. Assume that all the finalists are equally proficient spellers and that the outcome of the contest is random. What is the probability that 4 of the top 5 finishers will be female?

III.

A) State and prove the Law of Total Probability.

B) The Podunk City police department plans a crackdown on speeders by placing radar traps at four different locations L_1, L_2, L_3, L_4. The probability that each of the traps is manned at any one time is .4, .3, .2, .3 respectively (and the police really mean business – if a trap is manned every speeder who passes it will get a ticket). Speeders have probabilities of passing the four locations of .2, .1, .5, .2 respectively, and no one passes more than one. What is the probability that a given speeder will actually receive a ticket?

C) In the situation of part B, given that a speeder received a ticket, what is the probability that he passed location L_2?

IV. Let A, B be events for which $P(A) = .2$, $P(B) = .3$ and $P(A \cap B) = .06$. Are $\overline{A}$ and $\overline{B}$ independent events?

V. An allergist knows that 30% of all people are allergic to the pollen of burdock weed. The allergist sees 20 patients in all on one day.

A) What is the probability mass function for the random variable $Y =$ number of patients among the 20 who are allergic to burdock pollen? Explain the assumptions you are making to derive your solution.

B) What are the expected value and variance for the Y from part A?

C) What is the probability that from 4 to 7 (inclusive) of the 20 patients she sees are allergic to burdock pollen?

D) What is the probability that the first patient the doctor sees who is allergic to burdock pollen will be the last patient seen? Explain the assumptions you are making to derive your solution.

VI. *Extra Credit-Type Question (New)* The Yankees won 10 of the 18 regular season games they played against the Red Sox in 2009. Assume that the Red Sox and Yankees make it to the ALCS (a big assumption of course), and that the Yankees have a 10/18 chance of winning each game they play against the Sox (another *big* assumption at this point – recall that the Sox won the first 8 games and the Yankees won the next 10!). Let Y be the random variable that gives the number of the game in which the Yankees clinch the AL pennant (if they do), and 0 if the Red Sox pull out a win. What are the probability mass function, the expected value, and the variance of Y?