Mathematics 376 – Probability and Statistics II
Problem Set 5
due: February 24, 2006

A) (A follow-up problem to C from Lab Project 3.) Recall that in Lab Project 3, to determine the endpoints of the confidence interval for the ratio of variances σ_1^2/σ_2^2, you needed to work with a formula like this

$$P\left(\frac{S_1^2/S_2^2}{f_{\alpha/2}} \le \sigma_1^2/\sigma_2^2 \le \frac{S_1^2/S_2^2}{f_{1-\alpha/2}}\right) = 1 - \alpha.$$

In the problem from the Lab Project, you were looking for a 95% confidence interval, so you needed $f_{.025}, f_{.975}$. The F-table in the back of the book gives the values f_α for $\alpha = .1, .05, .025, .01, .005$, but not the complementary values $\alpha = .9, .95, .975, .99, .995$. As you might have wondered, *why aren't those other entries there??* Fortunately, with the `FCDF` function from our Maple package, you could compute $f_{.975}$, etc., by the same method we used for χ^2 and t distributions in the lab, so this wasn't a problem. In this question you will show that the other values could be derived from the table entries given with a little cleverness, too!

1) Let the numbers $f_\alpha(n, m)$ be defined by

$$P(Y \ge f_\alpha(n, m)) = \alpha$$

if Y has an F-distribution with n numerator degrees of freedom and m denominator degrees of freedom. Show that for all $0 < \alpha < 1$, these numbers satisfy:

$$f_{1-\alpha}(n, m) = \frac{1}{f_\alpha(m, n)}$$

(note the reversal in the numbers of degrees of freedom on the right.) **Hint:** What is the standard way to get a random variable that has an F-distribution?

2) Using the table in the text and part 1, determine the number $f_{.99}(7, 10)$.

Additional problems from the text:

- Chapter 8/74,75,82,86;