Mathematics 376 – Probability and Statistics II
Lab Day 2/Discussion – Confidence Intervals
February 13, 2006

Background

We have now introduced the "large-sample" confidence intervals for means and differences of means, proportions and differences of proportions. We use the point estimators from the table on page 371 to derive the confidence interval estimators.

For example, the large-sample level $(1-\alpha) \times 100\%$ two-sided confidence interval for a mean is

$$\mu = \overline{Y} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \tag{1}$$

When we don't know the population σ, we approximate with the sample standard deviation s.

Nevertheless, he concept of confidence intervals is a notoriously "slippery" one. It is easy to go from the intuitively appealing "confidence level" $(1-\alpha) \times 100\%$ to statements that seem to be saying the same thing, but are misleading at best, and completely meaningless at worst. To use this idea reliably to make inferences from real-world data, it is very important to understand *exactly* what it means. The purposes of today's lab/discussion are to

1) Reinforce the exact meaning of the confidence level $(1-\alpha) \times 100\%$ by sampling and analyzing the confidence intervals we get.

2) Look critically at some of the misleading or meaningless ways that the idea of a confidence interval can be "abused."

New Maple

Our class Maple statistics package contains a procedure called `CIPlot` which creates a graphic illustrating the meaning of confidence intervals. The procedure uses the formula (1) above (substituting $\sigma = s$ with each sample) to construct confidence intervals for the mean from samples from a normal population. Consult the online documentation for the Maple package (link on course homepage) to see how it works, and how to interpret the output. You can also examine the source code in the `MSP.map` file if you're interested.

Lab/Discussion Questions

(You will probably want to create a Maple worksheet containing the requested plots, plus answers to the questions below.)

1) Use the `CIPlot` procedure to generate the plot showing 100 95% confidence intervals for the mean, generated from samples of size $N = 35$ from a normal population with $\mu = 20$ and $\sigma = 4.5$.

2) Explain what your plot shows. In particular, how many of the intervals contain $\mu = 20$ and how many do not? How does that relate to the 95% confidence level?

3) Will the plot *always* show exactly 95 intervals containing the population mean $\mu = 20$? In what sense do 95% of the confidence intervals contain $\mu = 20$. (You may want to generate several more plots using `CIPlot` – each call uses new random samples, so the results will be different.)

4) (Try to answer this before going to Maple, then use the procedure to check your answer.) Suppose you now use `CIPlot` to study the 90% confidence intervals for the mean (i.e. new α), generated from samples of the same size $N = 35$ from the same normal population with $\mu = 20$ and $\sigma = 4.5$. What will change?

5) (Try to answer this before going to Maple, then use the procedure to check your answer.) Suppose you now use `CIPlot` to study the 95% confidence intervals for the mean (same α as in 1 above), generated from samples of the new size $N = 70$ from the same normal population with $\mu = 20$ and $\sigma = 4.5$. What will change (apart from the fact that the computation will take somewhat longer)? When you check your intuition, be sure to look carefully at the horizontal axis scales.

6) Suppose you now use `CIPlot` to study the 95% confidence intervals for the mean (same α as in 1 above), generated from samples of the new size $N = 10$ from the same normal population with $\mu = 20$ and $\sigma = 4.5$. Run the procedure 10 times and compute the total number of intervals that contain $\mu = 20$. (You only need to show the graph for the last one in your lab report.) The results here should be different from the other cases. Why is that true? (Recall that when we don't know σ, we approximate it by the sample standard deviation s. What is the exact distribution of $\frac{\overline{Y}-\mu}{s/\sqrt{N}}$? What do you conclude here?)

7) A common *misconception* about confidence intervals can be stated as follows: "When you compute the 95% confidence interval for the mean from a particular sample, there's a 95% chance that the population mean is contained in your interval." This statement (taken literally) is actually meaningless – why? What could you do to modify the statement so that it makes sense and is a true statement about confidence intervals?

8) Another common *misconception* about confidence intervals can be stated as follows: "When you increase the sample size N the width of the $(1 - \alpha) \times 100\%$ confidence interval always decreases." This statement is actually false – why? What could you do to modify the statement so that it makes sense and is a true statement about confidence intervals?

9) A final common (and tempting) *misconception* about confidence intervals deals with this situation. Say we are using the interval to decide whether evidence from samples

supports the hypothesis that a population mean μ has a particular value μ_0. If the value μ_0 is contained in a confidence interval but is far from the midpoint (even very close to one endpoint), it is tempting to think that the evidence indicates possibly $\mu \neq \mu_0$. On Friday, 2/10 we said that this is an *incorrect* deduction in this situation. However, we did not give a reason. Look carefully back at your results from part 1. Explain why concluding $\mu \neq \mu_0$ on the basis of one confidence interval where μ_0 is close to an endpoint is *not a correct conclusion.*

Assignment

Group writeups due in class Monday, February 20.