Mathematics 375 – Probability and Statistics 1

Problem Set 7 – *due*: Friday November 4

From the text: Chapter 4/73, 75, 77, 78, 86, 87, 92, 93.

Additional Problem

In most cases, the *only* way to compute probabilities with Gamma- and Beta-distributed random variables is to use numerical methods on the appropriate integrals of the pdf's. However, in some special cases, there are interesting connections with some of the discrete random variables we have studied before.

A) In this problem you will study a relation between continuous Gamma distributions and discrete Poisson distributions in the case that the parameter α of the Gamma distribution is a positive integer and $\beta = 1$.

1) Let α be a positive integer and consider a Gamma-distributed random variable Y with parameters α positive $\in \mathbf{Z}$, and $\beta = 1$. Let $\lambda > 0$ be arbitrary. Using integration by parts and a proof by mathematical induction, show that

$$\begin{aligned} P(Y \geq \lambda) &= \frac{1}{\Gamma(\alpha)} \int_{\lambda}^{\infty} y^{\alpha-1} e^{-y}\, dy \\ &= \sum_{n=0}^{\alpha-1} \frac{\lambda^n}{n!} e^{-\lambda} \end{aligned}$$

   (Note that the last line here is a sum of probabilities for a Poisson discrete random variable(!)).

2) Suppose Y has a Gamma distribution with $\alpha = 3$, $\beta = 1$. Using the result of part 1 and the tables in our text, find $P(Y \geq 5)$.