Mathematics 375 – Probability and Statistics 1
Information on Exam 2
November 9, 2005

General Information

The second midterm exam will be given either Wednesday evening November 16, or in class on Friday, November 18. The exam will cover the material we have discussed since the first exam, up to and including the material on Tchebysheff's Theorem and moment generating functions we discussed on Friday, November 4.

Format and Groundrules

This will be an *closed book* exam, but you may prepare *one side of a* 3×5 *inch index card* with formulas and any other information you want to include, bring it to the exam and consult it at any time. I will provide copies of the tables for Poisson and normal probabilities from the text. Calculators allowed.

Topics

The topics to be covered are:

1) The hypergeometric and Poisson discrete probability distributions – Sections 3.7 and 3.8.
2) Moment-generating functions for discrete random variables – Section 3.9.
3) Continuous random variables and probability distributions, cdfs and pdfs – Section 4.2
4) Expected values for continuous random variables – Section 4.3
5) Uniform, Normal, Gamma, and Beta distributions – Note that this includes the definition and properties of the Gamma and Beta functions. Be prepared for problems like those on the homework where you need to find the mean and variance of some *function of* a random variable of one of these types, and also for problems where you might use the geometric or binomial distributions in combination with one of these – Sections 4.4 - 4.7
6) Moment-generating functions for continuous random variables – Section 4.9
7) Tchebysheff's Theorem – Section 4.10

Review Session

I will be happy to run a pre-exam review session if there is interest. The evening of Monday, November 14 would be best if we do the exam on Wednesday, but other times might be possible too.

Suggested Review Problems

From the text: Chapter 3/157, 158, 160, 173, Chapter 4/67, 88, 101, 110, 128, 129, 52 and 131, 132, 133, 135, 136, 137, 142, 150, 151

Sample Exam

Disclaimer: The following problems represent the approximate length and level of difficulty of the upcoming exam; the actual exam problems may differ substantially from these.

Comments: Any general formula we have studied (or that you find in the text!) for mean, variance, etc. can be used without comment (i.e. you don't need to rederive it in you solution).

I. (15) Ten college students, exactly four of whom are not of legal age, go out to a bar and all order alcoholic beverages. The waitperson taking their order selects 5 students at random from the group to "card" and must refuse to serve any one who is under legal age. What is the probability that exactly 2 students will be refused service? (Assume, unrealistically perhaps, that each ID shows that student's actual age!)

II. The fraction Y of impurities in a tank-car lot of industrial grade potassium perchlorate is a random variable with probability density function

$$f(y) = \begin{cases} cy^2(1-y)^2 & \text{if } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

A) (10) What is the value of the constant c?
B) (10) What is the variance of Y?
C) (10) Find the cumulative distribution function for Y.
D) (10) Find $P(.1 \leq Y \leq .25)$.

III. The life in years of a certain type of electrical switch has an exponential distribution with mean 2 years.

A) (15) What is the probability that a single switch will fail during the first year after it is installed?
B) (5) If 100 of these switches are installed in different, independently operating systems, what is the probability that at most 30 fail during the first year? (You need *not* obtain a single decimal approximation to this number; a formula for computing it will suffice.)

IV. The weights of a population of miniature poodles are normally distributed with mean $\mu = 8$kg and standard deviation $\sigma = .9$kg.

A) (15) What is the probability that a randomly selected poodle will have weight between 7.3kg and 9.1kg?

B) (10) Let Y be the weight of a poodle from the population as a random variable. What is $E(Y^3)$? (Note: part credit will be given for a correct verbal description of a procedure that could be used to derive this value even if you cannot carry out the procedure completely.)

Extra Credit (10) Let Y be a random variable with probability density function

$$f(y) = \begin{cases} ye^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the moment-generating function $m_Y(t)$, assuming $t < 1$.