MATH 375 – Probability and Statistics 1
Discussion 1 – Counting
September 9, 2005

Background

We have discussed several techniques for counting different types of "arrangements" of elements from finite sets. These are useful for enumerating sample spaces in discrete probability problems. The key facts:

1) (the "mn rule") if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$. (Hence, we can count the number of elements in any set that can be put into one-to-one correspondence with the ordered pairs in $A \times B$.)
2) If $|A| = n$, then the number of *ordered lists* (or "permutations") of size $r \leq n$ chosen from A is
$$P^n_r = n(n-1)(n-2)\cdots(n-r+1).$$
3) If $|A| = n$, then the number of *unordered lists* (subsets, or "combinations") of size $r \leq n$ that can be chosen from A is
$$\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}.$$

In today's discussion, we will practice applying these to several different types of counting problems.

Discussion Questions

Note: In your answers, explain carefully how you are using one or more of the general patterns above to derive your results.

A) A standard deck of playing cards contains cards identified by a *suit* (clubs, diamonds, hearts, spades) and *denomination* $(2, 3, \ldots, 10$, jack, queen, king, ace).

1) Every possible combination of a suit and a denomination occurs exactly once in the deck (e.g. there is exactly one 10 of hearts). How many cards are there in a complete deck?
2) How many different 5-card hands can you be dealt in a game like poker (using a single standard deck)? How many different arrangements of cards are there if you are dealt a 5-card hand, then lay the cards down left to right in front of you? Explain.
3) In basic poker, the weakest type of hand that is "worth something" is a hand containing exactly one pair of cards of the same denomination (e.g. 2 8's), and three other cards, none of which matches the denomination of the pair (or each other!). How many different hands are there of this type? If you are dealt 5 cards randomly, what is the probability of getting one of these?

4) For the purposes of this question, a "flush" is a 5-card hand in which all cards have the same suit (e.g. a hand of 5 clubs). How many different hands are there of this type? (If you know what a "royal flush" is, note that they should be *included* in the hands you are counting here according to the definition of "flush" used here.)

B) Students attending BigState University can choose from 140 different major areas (Aardvark Husbandry to Zymurgy). A student's major is identified in the Registrar's computer data base by a two- or three-letter code (for example, the code for Cullinary Education is CE, while the code for Recreation Studies is RES). Double majors have just been approved by the faculty, and the Registrar wants to know whether it is possible to assign a unique code as above for every possible combination of one or two majors.

1) How many different possible double major programs are there? How many programs with a single or a double major?
2) How many different codes are there if all combinations of two or three capital letters are available?
3) Is the current system of codes "up to the job" of accounting for double majors?

C) The rugby team is conducting a raffle to raise money to buy new uniforms. In all, 1000 \$1 tickets are sold, including 25 tickets bought by the 25 team members (one apiece).

1) Suppose winners for three \$50 prizes are drawn randomly. How likely is it that at least two prizes are won by a team member?
2) Would your answer change if the prizes were \$100, \$35, and \$15? If so, how? If not, why not?

Assignment

Group writeups due Wednesday, September 14.