Mathematics 375 – Probability and Statistics 1
Information on Exam 1
September 28, 2005

General Information

As announced in the course syllabus, the first midterm exam will be given next week, either in class Friday, October 7. The exam will cover the material we have discussed since the beginning of the semester, up to and including the material on geometric discrete random variables from class on September 28. (This is the same as the material from Problem Sets 1 - 4 and Discussions 1 - 3). In more detail, the topics to be covered are:

1) Descriptive statistics such as the mean, standard deviation, and frequency histograms for numerical data, the "empirical rule"
2) Discrete sample spaces and counting techniques for sample points – the $m \cdot n$ rule, permutations, binomial and multinomial coefficients
3) The "Sample Point Method" for probabilities
4) Conditional probability and independence of events
5) The Law of Total Probability and Bayes' Rule – *know the proofs of these* and how to apply them.
6) The "Event Composition Method" for probabilities
7) Discrete random variables, probability distribution functions, expected value, variance – *know the proof of the equation* $V(Y) = E(Y^2) - (E(Y))^2$ *as well as how to apply it.*
8) Binomial and geometric random variables – *know the proof of the formula* $E(Y) = np$ *for a binomial random variable based on* n *trials with success probability* p, and the formulas for variance of binomial random variables, plus expected value and variance of geometric random variables.

Other Groundrules

You may prepare *one side of a* 3×5 *inch index card* with formulas and any other information you want to include, bring it to the exam and consult it at any time. I will provide copies of the tables for binomial random variables from the text. Calculators allowed.

Review Session

I will be happy to run a pre-exam review session if there is interest. The evening of Tuesday, October 4 would be just about the only evening I can be here that week, though, because of off-campus commitments.

Suggested Review Problems

From the text: 1.22, 1.25, 1.33, 2.122, 2.123, 2.124, 2.127, 2.129, 2.131, 2.132, 2.137, 2.144, 2.145, 3.15, 3.33, 3.53, 3.56, 3.61, 3.144

Sample Exam

This was the first midterm exam from the offering of this course in Fall 2003.

I. A manufacturer of electronic components tests the lifetimes of a certain type of battery and finds the following data:

$$123, 116, 122, 110, 125, 126, 111, 118, 117, 120$$

(lifetimes in hours). How many of the sample points are within one standard deviation of the sample mean? Is there reason to believe the lifetime of this type of battery is not normally distributed from this small sample? Explain.

II. In a regional spelling bee, the 10 finalists consist of 5 girls and 5 boys. Assume that all the finalists are equally proficient spellers and that the outcome of the contest is random. What is the probability that 4 of the top 5 finishers will be female?

III.

A) State and prove the Law of Total Probability.

B) The Podunk City police department plans a crackdown on speeders by placing radar traps at four different locations L_1, L_2, L_3, L_4. The probability that each of the traps is manned at any one time is .4, .3, .2, .3 respectively (and the police really mean business – if a trap is manned every speeder who passes it will get a ticket). Speeders have probabilities of passing the four locations of .2, .1, .5, .2 respectively, and no one passes more than one. What is the probability that a given speeder will actually receive a ticket?

C) In the situation of part B, given that a speeder received a ticket, what is the probability that he passed location L_2?

IV. Let A, B be events for which $P(A) = .2$, $P(B) = .3$ and $P(A \cap B) = .06$. Are $\overline{A}$ and $\overline{B}$ independent events?

V. An allergist knows that 30% of all people are allergic to the pollen of burdock weed.

A) Starting from some point in time, the first five patients in sequence that the allergist sees are not allergic to burdock pollen. Given that, what is the probability that the first patient the doctor sees who is allergic to burdock pollen will come after the 20th patient seen? Explain the assumptions you are making to derive your solution.

B) What is the probability that from 4 to 7 (inclusive) of the next 20 patients she sees are allergic to burdock pollen? Explain the assumptions you are making to derive your solution.

Extra Credit. The Yankees won 10 of the 19 regular season games they played against the Red Sox in 2003. Assuming that the Yankees have a 10/19 chance of winning each game they play against the Sox, as of last Tuesday (i.e. before the first game), what was the probability that the Sox will clinch the ALCS at home? Assume the games are independent events, and recall that the series starts with two games in New York, then moves to Boston for the next 3 (if needed), then back to New York for the remainder of the series (if needed).