Mathematics 375 – Probability and Statistics 1
Discussion 5 – The "Method of Moment-Generating Functions"
November 30, 2005

Background

Because of the Uniqueness Theorem, whenever we can compute the moment-generating function for a random variable and recognize it as one of our standard forms, then we know its distribution: that is, its probability density function, mean, variance, and hence "everything about it"(!) Today, we want to use this idea to work several examples and identify what we have.

Discussion Questions

A) First, let $Y_1, \ldots, Y_k$ be independent normally distributed random variables with respective means μ_i and variances σ_i^2. Consider a general linear combination:

$$Y = a_1 Y_1 + \cdots + a_k Y_k + b,$$

where a_i, b are constant. What is the distribution of Y? What are its mean and variance?

B) Next, we will verify one point that we deferred in discussing the use of the standard normal table. Recall, we said that if Y is normal with mean μ and standard deviation σ, then

$$Z = \frac{Y - \mu}{\sigma}$$

would have a standard normal distribution (i.e. normal with mean 0 and standard deviation 1). We never really justified this claim before. But we *can do it now*! Note that

$$Z = \frac{1}{\sigma} Y - \frac{\mu}{\sigma}.$$

Use the result of question A to deduce that Z must have a standard normal distribution.

C) Let Y be a standard normal, and let $X = Y^2$.

1) Set up the integral to compute

$$m_X(t) = E(e^{tX})$$

using the density for Y.

2) For the rest of the problem, assume $1 - 2t > 0$. Combine terms in your integral, make the substitution $u = \sqrt{1-2t} \cdot y$, and show that

$$m_X(t) = \frac{1}{\sqrt{1-2t}}.$$

3) What is the distribution of X? (What "type of random variable" is X, according to the Uniqueness Theorem?)

4) Suppose $Y_1, \ldots, Y_k$ are independent random variables, each with a standard normal distribution. What is the distribution of $X = Y_1^2 + \cdots + Y_k^2$?

Assignment

Group writeups due in class on Friday, December 2.