Mathematics 376 – Probability and Statistics 2
Review Sheet – Exam I
February 22, 2006

General Information

As announced in the course syllabus, the first midterm exam this semester will be given *Wednesday, March 1.* The exact scheduling will be determined in class on February 22. The exam will cover the material we have discussed since the start of the semester, up to and including the material on confidence intervals for variances and (this is section 7.2, and all of Chapter 8). There is a more detailed breakdown of topics given below.

Format

This will be an *closed book exam*, but you may place any information you think might be useful on one side of a standard 8.5×11 sheet of paper, bring that to the exam and consult it at any time. I will also provide copies of any tables from the text you will need to use. Even though you will have all that information to use, you should still prepare very carefully for the exam and understand the key concepts we have talked about. Know how to apply the different techniques we have studied. If you are consulting your "cheat sheet" for *every* formula you need or examples, etc. you *may not be able to finish the exam in the allotted time.*

Topics

The topics to be covered are:

1) Sampling distributions related to the normal distribution (χ^2, t, F distributions) – know how to tell when a random variable has one of these distributions, how to use the tables for each in the text, etc.
2) Point estimators for distribution parameters, bias, mean square error, standard error. Be sure you understand Table 8.1 on page 371 of the text, where all the entries come from, and how they are used. Also be able to analyze estimators to determine whether they are biased or not, construct unbiased estimators, etc. (see Problem Set 2).
3) The pdf's for order statistics (especially the sample maximum and minimum), and how they can be used for estimation problems – this was covered in class on February 1; also see section 6.7 in the text.
4) The concept of a confidence interval (interval estimator), derivation of formulas for interval estimators via pivotal quantities.
5) Large-sample confidence intervals for means, differences of means, proportions, differences of proportions (constructed using pivotal quantities that are normal)
6) Small-sample confidence intervals for means, differences of means (constructed using pivotal quantities that have t-distributions)
7) Confidence intervals for variances.

Comment: As you can tell, much of this depends heavily on the probability topics we learned last semester. If you are feeling "rusty" on general properties of expected values or variances like the important relations

$$E(aX + bY) = aE(X) + bE(Y)$$

and

$$V(aX + bY) = a^2V(X) + b^2V(Y) + 2abCov(X, Y),$$

or on computing expected values or variances of random variables with given distribution (i.e. given pdf), start by reviewing that material.

Review Session

I will be happy to run a pre-exam review session if there is interest. The evening of Monday, February 27 would be best, but other times might be possible too.

Suggested Review Problems

From the text:
Chapter 7/19,21,62,63,65,67,71;
Chapter 8/5,8,11,15,21,33,37,49,51,62,63,69,86

Sample Exam

Disclaimer: The following problems represent the approximate level of difficulty of the upcoming exam; the actual exam problems may differ substantially from these.

Comments: Any general formula we have studied can be used without comment (i.e. you don't need to rederive it in you solution).

I. Let $Z_1, Z_2, \ldots, Z_7$ be independent random samples from a standard normal distribution. Let

$$W = Z_1^2 + Z_2^2 + \cdots + Z_7^2$$

A) What is the distribution of W? Explain.
B) Find $P(1.68987 \leq W \leq 14.0671)$.
C) Let $U = Z_1^2 + Z_2^2 + \cdots + Z_6^2$ (not including Z_7). What is the distribution of $V = \frac{Z_7}{\sqrt{U/6}}$. Explain.
D) For which value of y is $P(V \geq y) = .05$?

II. A farm grows grapes for jelly. The following data are measurements of grape sugar levels from 8 random samples:

$$16.0, 15.2, 12.0, 16.9, 14.4, 16.3, 15.6, 12.9$$

Assume that these are independent samples from a normal distribution with parameters μ and σ^2.

A) Find point estimates μ and σ from the given information.

B) Construct a 90% confidence interval for the population mean μ from the given information. Explain how you know which method to use.

C) Construct a 95% confidence interval for the population variance σ^2.

III. Two machine shops manufacture toggle levers. A random sample of size 642 from the output of shop A produced 24 defective toggle levers. A random sample of size 500 from the output of shop B produced 22 defective toggle levers.

A) Give a point estimate of p_A, the proportion of defectives in the output of shop A.

B) Derive a 95% confidence interval for p_A.

C) Derive a 95% confidence interval the difference $p_A - p_B$ where p_B is the proportion of defectives in the output of shop B. Is there good reason to suppose the output of shop A has higher quality than the output of shop B? Explain.

IV. Let $Y_1, \ldots, Y_n$ (n fixed but arbitrary) be random samples from a uniform distribution on $[0, \theta]$. Let $\widehat{\theta} = Y_{(n)}$ (the sample maximum) be used as an estimator for θ. Show that $\widehat{\theta}$ is a biased estimator for θ, and find a constant c so that $c\widehat{\theta}$ is an unbiased estimator for θ.