

Mathematics 375 – Probability and Statistics 1
Information on Exam 2
November 13, 2003

General Information

As announced in the course syllabus, the second midterm exam will be given in class next *Friday, November 21*. The exam will cover the material we have discussed since the first exam, up to and including the material on Tchebysheff's Theorem we discussed on Friday, November 7.

Format

This will be an *open book, open class notes exam*. I'm doing that to try to minimize the pressure to memorize lots of stuff. But you should still prepare very carefully for the exam and understand the key concepts we have talked about. Know how to apply the different techniques we have studied. If you are flipping through the book to look for *every* formula you need or if you need to find examples similar to the test questions to get started on a problem, you *will not be able to finish the exam in 50 minutes*.

Topics

The topics to be covered are:

- 1) The hypergeometric and Poisson discrete probability distributions – Sections 3.7 and 3.8.
- 2) Moment-generating functions for discrete random variables – Section 3.9.
- 3) Continuous random variables and probability distributions, cdfs and pdfs – Section 4.2
- 4) Expected values for continuous random variables – Section 4.3
- 5) Uniform, Normal, Gamma, and Beta distributions – Note that this includes the definition and properties of the Gamma and Beta functions. Be prepared for problems like those on the homework where you need to find the mean and variance of some *function of* a random variable of one of these types, and also for problems where you might use the geometric or binomial distributions in combination with one of these – Sections 4.4 - 4.7
- 6) Moment-generating functions for continuous random variables – Section 4.9
- 7) Tchebysheff's Theorem – Section 4.10

Review Session

I will be happy to run a pre-exam review session if there is interest. The evening of Wednesday, November 19 would be best, but other times might be possible too.

Suggested Review Problems

From the text: Chapter 3/157, 158, 160, 173, 176cd, 178, Chapter 4/67, 88, 101, 110, 128, 129, 52 and 131, 132, 133, 135, 136, 137, 142, 150, 151

Sample Exam

Disclaimer: The following problems represent the approximate length and level of difficulty of the upcoming exam; the actual exam problems may differ substantially from these.

Comments: Any general formula we have studied (or that you find in the text!) for mean, variance, etc. can be used without comment (i.e. you don't need to rederive it in your solution).

I. In a shipment of 10 missiles, there are 3 defectives whose engines will not fire. Out of the shipment, 4 missiles are selected at random.

- A) What is the probability that all 4 will fire?
- B) What is the probability that at most 2 will not fire?

II. In a biomedical research experiment, the survival time Y in weeks of an animal exposed to a certain fatal disease is a random variable with a gamma distribution with $\alpha = 2$ and $\beta = 6$.

- A) What is the mean survival time of a randomly selected exposed animal?
- B) The cost of treating an exposed animal is $C = 10 + .1Y^2$. Find the mean and variance of C .
- C) If 6 exposed animals are chosen at random, what is the probability that at least two survive for 15 or more weeks?

III. The lifetime of a certain type of small motor is normally distributed with mean 10 years and standard deviation 2 years. The manufacturer replaces free all motors that fail while under warranty. But to avoid going out of business because of the cost of doing the free replacements, the manufacturer must set the warranty so that no more than 3% of the motors that fail will be covered. What should the length of the warranty be?

IV.

- A) Let Y be a binomial random variable based on n trials with success probability p . Find the moment-generating function of $X = aY + b$, where a, b are constant.
- B) Let Z be a standard normal random variable. What is the moment-generating function for $Y = Z^2$? What type of random variable is Y ? (Hint: You will want to assume that $1 - 2t > 0$ and use the substitution $u = \sqrt{1 - 2t} \cdot y$ to evaluate the integral for $E(e^{tY})$.)