Mathematics 375 – Probability and Statistics 1
Problem Set 3
due: In class, Friday September 26

A) Much of the mathematical theory of probability was first developed (ironically enough) to analyze what happens in *games of chance* including various card and dice games used for gambling. One famous example is the dice game "craps" (a mainstay of mobster movies, *The Sopranos*, etc.) In case you have never played, the rules of craps are the following:

1) At the start of the game, the player rolls two dice.
2) If the first total is 7 or 11, then the player wins immediately.
3) If the first total is 2,3, or 12, then the player loses immediately.
4) If the first total is anything else, that becomes the player's "point". The player rolls the dice again (repeatedly) until either the point total is obtained again, in which case the player wins, or else a 7 is rolled, in which case the player loses. *Any number of additional rolls is possible in this case; the game continues until either the point is rolled again, or a 7 is rolled.*

The Maple package for our class now contains a craps simulator procedure that can be used to generate any number of craps games and study the outcomes. *Note: I just added this yesterday (9/18/03), so you will need to download the package again to get a version including this new procedure.* After you have loaded the package, you call the procedure with a command of the form *Craps*$(n, verbose)$; where n is the number of games you want to simulate, and *verbose* (true or false) tells Maple how much output to print out as the games are played.

- *Craps(10,true);* – plays 10 full games and prints all rolls as it goes along. The final output is a list of 10 0's and 1's (0 = player loses in that game; 1 = player wins).
- *Craps(100,false);* – plays 100 full games and generates the final output, but doesn't show all the individual rolls.

1) Use the *Craps* procedure to play 5 games, print out all the rolls, and explain the outcome of each of the games.
2) Experiment with larger numbers of games (you'll probably want to do lots!). What can you say about the apparent *probability* that the player wins an individual game?
3) Now, we'll analyze the game using the probability concepts we have learned. We'll assume the dice used are "fair" (i.e. the probability of each possible number on each die is 1/6).
   a) What is the probability of winning immediately (on the first roll)? What is the probability of losing immediately?
   b) Now the tricky part. What is the probability of winning if your point was a 4 or a 10? a 5 or a 9? a 6 or an 8? (Recall, the game can go on for any number of rolls!)
   c) What is the total probability of winning? How well does this match your simulation results?

4) (**Extra Credit**) Biaggio "Buggsy" Squicciarini is a mobster who is addicted to craps, and he hates to lose. Thinking that it would be really good to roll lots of $3+4$ 7s, he arranges to have a "special" set of dice made where the probabilities for each possible roll are $P(1) = 1/8$, $P(2) = 1/8$, $P(3) = 1/4$, $P(4) = 1/4$, $P(5) = 1/8$, $P(6) = 1/8$. What is the probability that he will win a game of craps using these dice?

Note: Casino games are not likely to be popular if the player wins too rarely. Your answer for part 3 should give some evidence why "craps" is a popular game, but one that casino operators can make money on too(!)

From the text: 2.61, 63, 64, 66, 68, 69, 73, 80, 84, 87, 93, 99, 102, 107, 3.1,3