Mathematics 376 – Probability and Statistics II
Problem Set 6
Due: In class March 26, 2004

A) In this problem you will complete the derivation of the maximum likelihood estimators for the parameters μ and σ^2 of a normal distribution that we started in class on Wednesday, March 19. Recall that we saw the likelihood function is

$$L = L(y_1, \ldots, y_n \mid \mu, \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left(-\sum_{i=1}^{n}(y_i - \mu)^2/(2\sigma^2)\right)$$

so

$$\ln(L) = \frac{-n}{2}\ln(2\pi\sigma^2) - \sum_{i=1}^{n}(y_i - \mu)^2/(2\sigma^2)$$

We saw that

$$\frac{\partial \ln(L)}{\partial \mu} = \sum_{i=1}^{n}(y_i - \mu)/\sigma^2$$

and

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{\sum_{i=1}(y_i - \mu)^2/2}{(\sigma^2)^2}$$

Setting $\frac{\partial \ln(L)}{\partial \mu} = 0$, we got

$$\widehat{\mu}_{ML} = \overline{Y}$$

(the sample mean).

1) Substituting $\widehat{\mu}_{ML} = \overline{Y}$ for μ, solve the equation $\frac{\partial \ln(L)}{\partial \sigma^2} = 0$ for σ^2 to derive $\widehat{\sigma^2}_{ML}$. Is this estimator biased or unbiased?
2) Check that the values of $\widehat{\mu}_{ML}$ and $\widehat{\sigma^2}_{ML}$ are actually give a *maximum* (not just a critical point) of $\ln(L)$ using the second derivative test for functions of two variables.

From the text

9.78, 9.79, 9.92 (in part c, the efficiency is the ratio of the variances of the estimators – see Definition 9.1), 10.4, 10.7, 10.9, 10.23.