Mathematics 376 – Probability and Statistics II
Lab Project 3 – Hypothesis Testing
April 5 and 7, 2004

Background

We have now discussed various tests of hypotheses for means, variances, etc. of normal distributions. In this lab we will work through several realistic examples to illustrate the thinking process statisticians would use to select appropriate tests, and interpret the results.

New Version of Maple Package

Start by downloading the latest version of the Maple statistics package from the course homepage (version of March 16). There is a new graphical routine in this iteration that produces what are called "box-and-whisker" plots for lists of numerical data that you will want to use at several points in this lab. The idea is the following. To get a rough visual picture of the distribution of a data set, as an alternative to the relative frequency histogram, we can use a *box-and-whisker* plot. The idea of this graphical display is to show the locations of the *minimum*, the *25*th percentile, the median or *50*th percentile, the *75*th percentile and the maximum of the data values by drawing a box with vertical bars at the 25th, 50th, 75th percentile values, together with two thinner "whiskers" extending out to the minimum and maximum. The new procedure in the Maple package is designed to draw one of these plots for any collection of lists, "stacked vertically," in one graphical display so that we can visually compare data sets in a rough way. Check the on-line documentation for more information and a usage example.

Lab Questions

A) An office furniture manufacturer has developed a new glue application process for assembling tables. To compare the two processes, random samples with $n = 30$ were selected from inventories produced with the old process and with the new process. Each table was subjected to "destructive testing" in which the force (in pounds) needed to break the glue in the table was measured. Let X be the force in pounds needed to break one of the new tables, and Y be the force in pounds needed to break one of the old tables. The goal is to determine whether the new gluing process has significantly increased the strength of the tables. The data collected was as follows:

	1250	1210	990	1310	1320	1200	1290	1360	1200	1150
X :	1120	1360	1310	1110	1320	980	950	1430	1100	1080
	960	1050	1310	1240	1420	1170	1470	1060	1230	1300

and

	1180	1360	1310	1190	920	1060	1440	1010	1000	950
Y :	1310	980	1310	1030	960	800	1280	1080	900	1030
	930	1050	1010	1310	940	860	1450	1070	840	1100

1) Construct box-and-whisker plots for these data sets (plotted together) and make an informal conjecture about whether or not the new process (the X data) has increased the strength of the tables, compared with the Y data.
2) Describe an appropriate test of the null hypothesis $H_0 : \mu_X = \mu_Y$ versus $H_a : \mu_X > \mu_Y$. Say what your assumptions about the data are, what your test statistic is, what the rejection region will be, and so forth.
3) Carry out your test at the $\alpha = .05$ level of significance. Give a clear and concise statement of the conclusion you draw from your test.
4) What is the attained significance level of your test (the p-value)? Use the appropriate CDF function in the Maple package to determine an accurate estimate of p, not just a range of possible values. What does this say?

B) Let X be the weight in grams of a Low-Fat Strawberry Kudo bar and let Y be the weight in grams of a Low-Fat Blueberry Kudo bar. Assume that the distributions of X, Y are normal: $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$ respectively. A random sample of $n_X = 9$ X weights were taken:

$$21.7, 21.0, 21.2, 20.7, 20.4, 21.9, 20.2, 21.6, 20.6$$

Similarly, a random sample of $n_Y = 13$ observations of Y were made:

$$21.5, 20.5, 20.3, 21.6, 21.7, 21.3, 23.0.21.3, 18.9, 20.0, 20.4, 20.8, 20.3$$

Random variations in the manufacturing processes mean that the weights of the bars are not always the same, and appear to vary depending on the type of bar as well. Is there a statistically demonstrable difference in the weight distributions of two types of bars, though? Here is one possible procedure for testing for equality of two normal distributions in the small sample case:

- First, test for equality of variances.
- If there is no demonstrable difference in the variances, test for equality of the means using the t-test for equality of means that we discussed in class (using the pooled estimator S_p for the common variance).
- If there is a demonstrable difference in the variances, the basic t-test is not all that reliable in many cases. With small sample sizes, most experienced statisticians would use a different approximating distribution due to Welch: Test for equality of means using a t-distribution with r degrees of freedom where

$$r = \left[\frac{(S_X^2/n_X + S_Y^2/n_Y)^2}{(S_X^2/n_X)^2/(n_X - 1) + (S_Y^2/n_Y)^2/(n_Y - 1)} \right] \tag{1}$$

(where $[z]$ = greatest integer less than or equal to z).

In this problem, you will follow this procedure to test this data:

1) Test the null hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$ against the alternative hypothesis $H_a : \sigma_X^2 \neq \sigma_Y^2$. You may select the significance level α. Interpret your results.

2) Test the null hypothesis $H_0 : \mu_X = \mu_Y$ against the alternative hypothesis $H_a : \mu_X^2 \neq \mu_Y^2$. Select the test statistic, etc. based on the results of your test from part 1 and explain your choice. Also clearly state the conclusion you draw from the test.
3) Construct box-and-whisker plots of the two data sets and reconcile with your results in parts 1 and 2.

Assignment

Group write-ups due *at the end of the class on Wednesday, April 7.*