Mathematics 375 – Probability and Statistics II
Review Sheet – Exam I
February 18, 2004

General Information

As announced in the course syllabus, the first midterm exam this semester will be given in class on *Friday, February 27.* The exam will cover the material we have discussed since the start of the semester, up to and including the material on small-sample confidence intervals for means (using t-distribution) from class on February 20 (this is all of Chapter 7, and sections 1-8 of Chapter 8. There is a more detailed breakdown of topics given below.

Format

This will be an *open book, open class notes exam.* But you should still prepare very carefully for the exam and understand the key concepts we have talked about. Know how to apply the different techniques we have studied. If you are flipping through the book to look for *every* formula you need or if you need to find examples similar to the test questions to get started on a problem, you *will not be able to finish the exam in 50 minutes.*

Topics

The topics to be covered are:

1) Sampling distributions related to the normal distribution (χ^2, t, F distributions) – know how to tell when a random variable has one of these distributions, how to use the tables for each in the text, etc.
2) the Central Limit Theorem, and the moment-generating function technique we used to prove it
3) The normal approximation to binomial distributions
4) Point estimators for distribution parameters, bias, mean square error, standard error. Be sure you understand Table 8.1 on page 371 of the text, where all the entries come from, and how they are used. Also be able to analyze estimators to determine whether they are biased or not, construct unbiased estimators, etc. (see Problem Set 3).
5) The pdf's for order statistics (especially the sample maximum and minimum), and how they can be used for estimation problems – this was covered in class on February 11; also see section 6.7 in the text.
6) Large-sample confidence intervals for means, differences of means, fractions, differences of fractions (constructed using pivotal quantities that are normal)
7) Small-sample confidence intervals for means, differences of means (constructed using pivotal quantities that have t-distributions)

Comment: As you can tell, much of this depends heavily on the probability topics we learned last semester. If you are feeling "rusty" on general properties of expected values or variances like the important relations

$$E(aX + bY) = aE(X) + bE(Y)$$

and

$$V(aX + bY) = a^2V(X) + b^2V(Y) + 2abCov(X, Y),$$

or on computing expected values or variances of random variables with given distribution (i.e. given pdf), start by reviewing that material.

Review Session

I will be happy to run a pre-exam review session if there is interest. The evening of Wednesday, February 25 would be best, but other times might be possible too.

Suggested Review Problems

From the text: Chapter 7/19,21,23,29,55,62,63,65,67,71; Chapter 8/6,8,11,15,21,33, 37,49,51,62,63,69

Sample Exam

Disclaimer: The following problems represent the approximate length and level of difficulty of the upcoming exam; the actual exam problems may differ substantially from these.

Comments: Any general formula we have studied can be used without comment (i.e. you don't need to rederive it in you solution).

I. Let $Z_1, Z_2, \ldots, Z_7$ be independent random samples from a standard normal distribution. Let

$$W = Z_1^2 + Z_2^2 + \cdots + Z_7^2$$

A) What is the distribution of W? Explain.
B) Find $P(1.68987 \le W \le 14.0671)$. (Note: This can be done by "brute force" using the pdf corresponding to the correct answer to part A, or by using the appropriate table in the text based on the correct answer to part A. The brute force way is a *much larger* computation – you probably don't want to try to do it that way on an exam, but setting up what you would have to do would earn substantial part credit if you cannot see how to get the desired info from the table!)

II. Let $\overline{X}$ be the mean of a random sample of size $n = 36$ from an exponential distribution with $\beta = 3$.
A) What is $P(2.5 \le \overline{X} \le 4)$?
B) Explain the general fact are you using to derive your solution to part A.

III. A farm grows grapes for jelly. The following data are measurements of grape sugar levels from 8 random samples:

$$16.0, 15.2, 12.0, 16.9, 14.4, 16.3, 15.6, 12.9$$

Assume that these are independent samples from a normal distribution with parameters μ and σ.

A) Estimate μ and σ from the given information.

B) Construct a 90% confidence interval for μ from the given information. Explain how you know which method to use.

IV. A machine shop manufactures toggle levers. Let p be the fraction of defectives in the shop's output. A random sample of size 642 produced 24 defective toggle levers.

A) Give a point estimate of p.

B) Derive a 95% confidence interval for p.

V. Let $Y_1, \ldots, Y_n$ be random samples from a uniform distribution on $[0, \theta]$. Let $\widehat{\theta} = Y_{(n)}$ (the sample maximum) be used as an estimator for θ. Show that $\widehat{\theta}$ is a biased estimator for θ, and find a constant c so that $c\widehat{\theta}$ is an unbiased estimator for θ.