MATH 242 – Principles of Analysis
Problem Set 1 – due: Jan. 31

'A' Section

1. Assume that A, B are sets of integers.
   a. What is the contrapositive of the statement: "If x is even then $x \in A \cup B$"? Express without using *not*.
   b. What is the converse of the statement in part a?
2. Let $A = \{x \in \mathbf{R} \mid x^2 - 5x + 4 = 0\}$, $B = (0, 1) = \{x \in \mathbf{R} \mid 0 < x < 1\}$ and $C = \{\frac{x}{x^2+9} \mid x \in \mathbf{R}\}$ (Note: C is the range of the function f defined by $f(x) = \frac{x}{x^2+9}$.)
   a. Express the set C as a closed interval $[a, b]$ in $\mathbf{R}$ or as a union of such intervals. (Note: You should use facts from calculus to solve this. Don't worry that we have not justified them yet.)
   b. Find the sets $B \cap A$ and $B \cap C$.
   c. Find the sets $A \cup B$ and $A \cup C$ and express using set notation.
3. For n a general natural number, let $B_n = \{0, 2n\}$. What are $\cap_{n=1}^{\infty} B_n$ and $\cup_{n=1}^{\infty} B_n$?
4. Let $I_n = [-1/n, 1/n]$ for a general natural number $n \geq 1$. What are $\cap_{n=1}^{\infty} I_n$ and $\cup_{n=1}^{\infty} I_n$? (Explain your reasoning intuitively.)
5. Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = \tan^{-1}(x)$.
   a. Is f one-to-one? Why or why not?
   b. Is f onto? Why or why not?
   c. If $I = (0, \sqrt{3})$, what is the set $f(I)$? Explain.
   d. If $J = (-\pi/4, \pi/4)$, what is the set $f^{-1}(J)$. Explain.

'B' Section

1. Prove part (f) of Theorem 1.1.3 in the text. These are the *DeMorgan Laws* for unions and intersections.
2. Let A and B be arbitrary sets. Does $A = A - (B - B)$, as we might expect if we looked at the formula through the lens of ordinary algebra? If this is always true, prove it; if it is not, give both a counterexample (an example where the formula is not true).
3. Let $f : A \to B$ be a function.
   a. Let C, D be subsets of A. Is it always true that $f(C \cap D) = f(C) \cap f(D)$? If this is always true prove it; if it is not, give a counterexample.
   b. Show that f is one-to-one if and only if $f^{-1}(f(C)) = C$ for all subsets C of A.
4. Let $f : A \to B$ and $g : B \to C$.

a. Show that if f and g are both onto, then $g \circ f : A \to C$ is also onto.
b. Is the converse of the statement in part a true? That is, if you know that $g \circ f$ is onto, does it follow that f and g are onto? Prove or find a counterexample.