MATH 242 – Principles of Analysis
Problem Set 1 – due: Feb. 1

'A' Section

1. Let $A = \{x \in \mathbf{R} \mid x^2 - 5x + 6 = 0\}$, $B = (0, 4) = \{x \in \mathbf{R} \mid 0 < x < 4\}$ and $C = \{\frac{x}{x^2+1} \mid x \in \mathbf{R}\}$ (Note: C is the range of the function f defined by $f(x) = \frac{x}{x^2+1}$.)
   a. Express the set C as a union of one or more closed intervals $[a, b]$ in $\mathbf{R}$. (Note: You should use facts from calculus to solve this. Don't worry that we have not justified them yet.)
   b. Find the sets $A \cap C$ and $B \cap C$.
   c. Find the sets $B \cup A$ and $B \cup C$ and express as unions of intervals in $\mathbf{R}$.
2. Let $B_n = \{1, 1/4, 1/9, \ldots, 1/n^2\}$ for each natural number $n \geq 1$. What are $\cap_{n=1}^{\infty} B_n$ and $\cup_{n=1}^{\infty} B_n$.
3. Let $I_n = [-1/n, 1/n]$ for any natural number $n \geq 1$. What are $\cap_{n=1}^{\infty} I_n$ and $\cup_{n=1}^{\infty} I_n$? (Explain your reasoning intuitively.)
4. Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = x^2 - 4x + 1$.
   a. Is f one-to-one? Why or why not?
   b. Is f onto? Why or why not?
   c. If $I = (1, 3)$, what is the set $f(I)$? Explain.
   d. If $J = (5, 6)$, what is the set $f^{-1}(J)$. Explain.

'B' Section

1. Prove part (f) of Theorem 1.1.3 in the text. These are the *De Morgan Laws* for complements.
2. Let A and B be arbitrary sets. Does $B = A - (A - B)$, as we might expect if we looked at the formula through the lens of ordinary algebra? If this is always true, prove it; if it is not, give both a counterexample (an example where the formula is not true), and a correct statement with proof.
3. Let $f : A \to B$ be a function.
   a. Let C, D be subsets of A. Is it always true that $f(C \cup D) = f(C) \cup f(D)$? If this is always true prove it; if it is not, give a counterexample.
   b. Show that f is onto if and only if $f(f^{-1}(E)) = E$ for all subsets E of B.
4. Let $f : A \to B$ and $g : B \to C$.
   a. Show that if f and g are both one-to-one, then $g \circ f : A \to C$ is also one-to-one.

b. Is the converse of the statement in part a true? That is, if you know that $g \circ f$ is one-to-one, does it follow that f and g are one-to-one? Prove or find a counterexample.