MATH 242 – Principles of Analysis
Exam 2 – April 1, 2011

Directions

Do all work in the blue exam booklet. There are 100 possible regular points and 10 Extra Credit Points.

I.

A) (20) State and prove the Monotone Convergence Theorem for sequences. (You may give the proof in the case that the sequence is monotone increasing.)

B) (10) Suppose that x_n is a sequence of strictly *negative* numbers and $x_{n+1}/x_n \leq 1$ for all $n \geq 1$. Show that $\lim_{n\to\infty} x_n = a$ exists in $\mathbf{R}$ and satisfies $a \leq 0$.

II. (20) Suppose $\{x_n\}$ is a sequence such that $|x_n - 10| < 100$ for all $n \geq 1$. Show that there exists some number $a \in [-90, 110]$ and a subsequence $\{x_{n_k}\}$ such that $x_{n_k} \to a$. State any "big theorems" you are using.

III.

A) (10) Show using the ε, δ definition that $\lim_{x\to 2} x^2 - x = 2$.

B) (10) Show using the ε, n_0 definition that $\lim_{n\to\infty} \frac{(-1)^n}{n^2+1} = 0$.

IV. True/False. For each true statement, give a short proof. For each false statement, give a counterexample or reason.

A) (10) If $\{x_n\}$ diverges, then for all $a \in \mathbf{R}$, there exist $\varepsilon_0 > 0$ and $n_0 \in \mathbf{N}$ such that $|x_n - a| \geq \varepsilon_0$ for all $n \geq n_0$.

B) (10) It is possible to find a function $f : \mathbf{R} \to \mathbf{R}$ that is continuous at $c = e$, with $f(e) = 0$, but satisfying $f(x) = 1$ for all rational x.

C) (10) The sequence defined by $x_1 = 1$ and $x_n = \sqrt{3x_{n-1} + 1}$ for $n \geq 2$ is monotone increasing.

Extra Credit. (10)

Assume that $\{x_n\}$ is a sequence that converges to a. Construct a new sequence $\{y_n\}$ by making y_n the average of the first n terms in the original sequence: $y_n = \frac{x_1+x_2+\cdots+x_n}{n}$. Show that $\{y_n\}$ also converges to a.