Mathematics 242 – Principles of Analysis
Problem Set 8, *Due:* 11/18

I. From Abbott: 4.5.2, 4.5.3 (of course, you are supposed to *prove* your assertion!), 4.5.7, 5.2.3, 5.2.4.

Additional Problems

II.

A) Show that if $f(x)$ is any polynomial of *odd* degree, then f has a real root (that is a real number c where $f(c) = 0$).

B) Suppose that $f(x)$ is a polynomial of *even* degree and that there exist $a \neq b$ in $\mathbf{R}$ such that $f(a) < 0 < f(b)$. Show that f has at least two distinct real roots.

C) True or False: Any polynomial of even degree with a real root has at least two distinct real roots.

III. Show that the equation $x^2 = 3^x$ has at least one real solution.

IV.

A) Suppose f is continuous on $[0, 1]$ with $f(0) < 0$ and $f(1) > 1$. Prove that there is at least one point $c \in (0, 1)$ where $f(c) = c^2$.

B) Generalize your reasoning from part A to show that if f is as before and g is any continuous function on $[0, 1]$ with $g(0) \geq 0$, $g(1) \leq 1$, the there is at least one point $c \in (0, 1)$ where $f(c) = g(c)$.