Mathematics 242  Principles of Analysis
Problem Set 5, Due: 10/14

I. Decide whether or not each of the following infinite series converges, and justify your claim using one of the results in sections 2.4 and 2.7 of Abbott:

A) \[
\sum_{n=0}^{\infty} \frac{1}{\sqrt{2n + 1}}
\]

B) \[
\sum_{n=0}^{\infty} \frac{1}{(2n + 1)^2}
\]

C) \[
\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^{2n}}
\]

D) \[
\sum_{n=0}^{\infty} (-1)^n \frac{n^3 + n}{2n^3 + n + 1}
\]

II.
A) For which \(x \in [0, 2\pi]\) does the series

\[
\sum_{n=1}^{\infty} \sin^n(x)
\]

converge? What is the sum for those \(x\)?

B) For which \(x \in \mathbb{R}\) does the series

\[
\sum_{n=1}^{\infty} e^{nx}
\]

converge? What is the sum for those \(x\)?

III. From Abbott: 2.7.3, 2.7.4, 2.7.9.

IV. Use the Ratio Test (Exercise 2.7.9 above) to determine for which \(x \in \mathbb{R}\) the series

\[
\sum_{n=0}^{\infty} \frac{(x - 1)^n}{3^n}
\]

converges. (Hint: The Ratio Test will give an open interval of values of \(x\) on which the series converges absolutely. What happens at the endpoints of that interval?)