Mathematics 242 – Principles of Analysis
Discussion 1 – Sets and Functions
September 2, 2005

Background

Today, we will review (or learn) some language about sets and functions. To begin, for us, a *set* will be a collection of objects, called the *elements* of the set. Usually, but not always, the elements of a set will be mathematical "things" like numbers, geometric figures, and so forth. Usually sets are named by capital letters like A, B, X, Y, and elements are named a, b, x, y, etc. The statement that x is an element of the set A is written in symbolic form as $x \in A$ (the symbol $\in$ means "is an element of"). We can specify sets by either

- listing the elements,
- giving a "recipe" for constructing all the elements, or
- giving a "test" for whether an element of a larger "universal" set is contained in the set we are talking about.

For instance, consider the set of natural numbers $S = \{1, 3, 5, 7, 9\}$. We could also define the same set as

$$S = \{2k + 1 : 0 \leq k \leq 4, k \in \mathbf{N}\}$$

or as

$$S = \{n \in \mathbf{N} : n \text{ is odd and } n \leq 9\}$$

(among many other ways). Today, we will look at a number of examples.

Discussion Questions

A) Given sets A, B inside some universal set X, we can make other sets called the *union* $A \cup B$, the *intersection* $A \cap B$, and the *complement* A^c (complement inside X). How are each of these defined? If you don't know or don't remember, look in section 1.2 of the text, and explain what is there in your own words.

B) Let the universal set be $X = \mathbf{Z}$ (the set of integers) and let

$$A = \{n \in \mathbf{Z} : n^2 < 12\} \qquad B = \{n \in \mathbf{Z} : 1 < 2n + 5 < 20\}$$

1) List the elements in A, B.
2) What are the sets $A \cap B$, $A \cup B$ and A^c?
3) Show that $(A \cup B)^c$ is the same set as $A^c \cap B^c$ by describing the elements of each.

C) For each natural number n, let $A_n = \{x \in \mathbf{R} : x \geq n\}$. (We could also write the set A_n using interval notation as $A_n = [n, +\infty)$.) Determine the following sets:

1) $A_1 \cap A_2$, $A_1 \cap A_2 \cap A_3$, $A_1 \cap A_2 \cap A_3 \cap A_4$.

2) What set is the intersection of *all* the A_n's:

$$\bigcap_{n=1}^{\infty} A_n = A_1 \cap A_2 \cap A_3 \cap \cdots ?$$

Explain.

D) Let A, B be sets. A *function* $f : A \to B$ is a rule or mapping that to each element $x \in A$ assigns a single element $f(x) \in B$. The set A is called the domain of f. The set of elements $f(A) = \{y \in B : y = f(x) \text{ for some} x \in A\}$ is called the range of f.

1) Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = x^2 + 1$. What is the range of f?
2) Same question for $f : [-1, 1] \to \mathbf{R}$ defined by $f(x) = \arctan(x)$. (Note the domain!)
3) How would you define the *graph* of a function using set notation? (What new type of object is involved?)
4) Consider the function $f : \mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbf{Q} \\ 0 & \text{if } x \notin \mathbf{Q} \end{cases}$$

Draw (or describe in words) the graph of this function.

Assignment

Writeups due in class on Monday, September 5.