Mathematics 242 – Principles of Analysis
Information on Final Examination
December 6, 2004

*General Information*

- The final examination for this class will be given during the scheduled period – 8:30 to 11:30 am on Wednesday, December 15.
- The final will be a *comprehensive exam*, covering all the topics from the three midterms, *and the material about infinite series from the past week.* See the list of topics below for more details.
- The exam will be similar in format to the midterms but roughly twice as long – it will written to take about 2 hours if you work steadily, but you will have the full 3 hour period to use if you need that much time.
- If there is interest, I would be happy to arrange an evening review session during exam week (Monday would probably be the best day). We can discuss this in class on December 6.

*Philosophical Comments and Suggestions on How to Prepare*

- The reason we give final exams in almost all mathematics classes is to encourage students to "put whole courses together" in their minds. Also, preparing for the final should help to make the ideas "stick" so you will have the material at your disposal to use in later courses.
- If you approach preparing for a final exam in the right way it can be a *real learning experience* – especially in a class like this one where almost everything we have done "fits together" in a very tight chain of logical reasoning starting with the Completeness Axiom for the real number system. Much of what we did earlier in the semester may and should make much more sense now than it may have the first time around!
- *Start reviewing now*, and do some review each day between now and December 15 (even just 1/2 hour each day will make a big difference). That way you will not be "crunched" at the end (and with any luck the ideas we have developed in this course will "stick" better!)

*Topics To Be Included*

0) Logic, sets, functions and relations
1) The real number system, rational and irrational numbers, the algebraic and order properties, the least upper bound (completeness) axiom.
2) Mathematical induction
3) Sequences, convergence ($\lim_{n\to\infty} x_n$ – both $\varepsilon$ - $N$ definition, and computing limits via the limit theorems.
4) Subsequences, The Nested Interval Theorem, and the Bolzano-Weierstrass theorem
5) Limits of functions, the limit theorems (sum, product, quotient, "squeeze" rules), one-sided limits

6) Continuity, the Extreme and Intermediate Value Theorems
7) Definition and properties of the derivative, the Mean Value Theorem and its consequences
8) The definite integral, integrability, the Fundamental Theorem
9) Infinite series – convergence and divergence, key examples such as geometric series, $p$-series, etc. Absolute vs. conditional convergence. Comparison, integral, alternating series, and ratio tests for convergence.

*Proofs to Know*

You should be able to give precise statements of *all the definitions* listed on the course homepage and the theorems mentioned in the outline above. Also, be able to give proofs of the following:

1) Every monotone increasing sequence of real numbers that is bounded above converges.
2) The Intermediate Value Theorem (proof we did in class relying on Bolzano-Weierstrass theorem – Note: this is *different* from the proof given in the text in Section 22)
3) The Mean Value Theorem
4) The Fundamental Theorem of Calculus

*Suggested Review Problems*

See review sheets for Midterm Exams 1, 2, and 3 for topics 1-8 in the list above. (Those review sheets are now reposted on the course homepage if you need another copy.) For topic 9:

Section 32/4, 5, 7
Section 33/3, 4, 5