Plato's Criticism of Geometers of His Time
From Plutarch, *Quaestiones Convivales*, Book 8, Chapter 2, Section 1
Text for Oral Presentation

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GREK 338 – Plutarch Seminar
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December 6, 2016

The Plutarch passage I will discuss comes from a section of the *Moralia* (Ἠθικά) known as the Συμποσιακά, or "Table Talk." Each section of this work is presented as a record of conversation at a συμπόσιον or drinking party arranged by Plutarch for a group of guests. Each of these parties involved consideration of some philosophical question and it is amusing to see what Plutarch says about the rationale for this: in "... our entertainments we should use learned and philosophical discourse ..." so that even if the guests become drunk, "... every thing that is brutish and outrageous in it [i.e. drunkenness!] is concealed ... ." In other words, to keep your next party from degenerating into a drunken brawl, have your guests converse about Plato!

During a celebration of Plato's birthday, Plutarch presents a conversation on the role of geometry in Plato's thought. One of the guests begins by asking why Plato asserted that "God is always doing geometry." A second guest replies that Plato wrote that geometry "takes us away from the sensible and turns us back to the eternal nature we can perceive with our minds, whose contemplation is the goal of philosophy... ." He then presents this interesting piece of evidence in our Greek text:

"δίο καὶ Πλάτων αὐτὸς ἐμέμψατο τοὺς περὶ Εὔδοξον καὶ Ἀρχύταν καὶ Μέναιχμον εἰς ὀργανικὰς καὶ μηχανικὰς κατασκευὰς τὸν τοῦ στερεοῦ διπλασιασμὸν ἀπαγεῖν ἐπιχειροῦντας ... ." "Therefore even Plato himself found fault with Eudoxus, Archytas, and Menaechmus for attempting to reduce the *duplication of the cube* to mechanical constructions with instruments ... . "

Now, the *duplication of the cube* asked for the construction of the side of a cube whose volume would be twice the volume of a given cube. This problem came to be associated with a reputed pronouncement of the oracle at Delphi to the people of the island of Delos. They were instructed to construct an altar twice as large as an existing one to stop the progress of a plague on their island. Elsewhere in the *Moralia*, Plutarch says the oracle's agenda was actually to encourage the Greeks to pursue the study of geometry more diligently. If so, the commandment was definitely successful–this was one of a series of construction problems that stimulated much of the development of Greek geometry through the Classical period. Eudoxus [of Cnidus (409–356 BCE)], Archytas [of Tarentum (428–347 BCE)], and Menaechmus [of Alopeconnesus (380–320 BCE)] were three of the most accomplished Greek mathematicians active in the 4th century BCE. Archytas is often identified as a Pythagorean and there are traditions that Eudoxus was a pupil of his and Menaechmus was a pupil of Eudoxus, all three being associated with Plato and his Academy in Athens in some way.

The second speaker continues in a surprisingly technical vein,
"ὥσπερ πειρωμένους δι᾽ ἀλόγου δύο μέσας ἀνάλογον, ᾗ παρείκοι, λαβεῖν ... ". This phrase has caused much perplexity (one editor proposes removing it entirely) and many variant readings. As mathematics, though, this is a key point: "just as though they were trying, in an unreasoning way, to take two mean proportionals in continued proportion any way that they might ... ". [In the Plutarch passage, the "two mean proportionals" appears in the accusative as δύο μέσας ἀνάλογον. The ἀνάλογον seems to be essentially equivalent to ἀνὰ λόγον – "in (continued) proportion."] It is interesting that the speaker seems to be assuming that all of his listeners will be familiar with this terminology and a key episode in the history of Greek geometry.

A key piece of progress on the duplication of the cube was made somewhat before the time of Plato by Hippocrates of Chios (ca. 470–ca. 410 BCE).

Given two line segments AB and GH, we say line segments CD and EF are *two mean proportionals in continued proportion* between AB and GH if their lengths satisfy:

$$\frac{AB}{CD} = \frac{CD}{EF} = \frac{EF}{GH}. \tag{1}$$

If in addition $GH = 2AB$, then some simple algebra [sketched on the handout] shows

$$CD^3 = 2AB^3.$$

In other words, if AB is the side of the original cube, then CD is the side of the cube with twice the volume. This was not a full solution, but it did provide a way to attack the problem and almost all later work took this as its starting point.

The speaker concludes his summary of Plato's criticism by claiming that [solving the problem that way] "ἀπόλλυσθαι γὰρ οὕτω καί διαφείρεσθαι τὸ γεωμετρίας ἀγαθὸν αὖθις ἐπὶ τὰ αἰσθητὰ παλινδρομούσης καὶ μὴ φερομένης ἄνω μηδ΄ ἀντιλαμβανομένης τῶν ἀίδιων εἰκόνων πρὸς αἷσπερ ὢν ὁ θεὸς ἀεὶ θεός ἐστι."
"... in this way the good of geometry is utterly destroyed and it falls back on the senses; it is neither carried above, nor apprehends the eternal and immaterial images (forms), before which God is always God." That is, by their nature, the solutions proposed by Eudoxus, Archytas, and Menaechmus subverted what Plato saw as the true purpose of geometry, which was not merely to solve problems "by any means necessary," but rather to lead the soul to the contemplation of eternal truth.

Plutarch does not include any discussion of what Eudoxus, Archytas, or Menaechmus actually did. However, such information has survived in an ancient source: a much later commentary on Archimedes' *On the Sphere and*

Cylinder by Eutocius [of Ascalon (ca. 480 – ca. 540 CE)]. It's interesting (but also very subtle) to try to see to what extent the adjectives μηχνικός or ὀργανικός from Plato's supposed criticism apply. If anyone is interested in the details, I would be happy to share a fuller write-up.

Here's the briefest of summaries: On the face of it, ὀργανικός, in the sense of instrument-based, or tool-based, is clearer. For ὀργανικός to apply, I believe some physical device must be involved in the construction. But there is a subtle point. Even though they certainly used physical straightedges to draw lines and physical compasses to draw circles while constructing diagrams, Greek geometers considered those tools in *idealized versions* (e.g. in the first three Postulates in Book I of Euclid's *Elements*) as purely mental constructs. Plato probably would have had no "beef" with the use of those tools. Eutocius discusses much more elaborate physical devices for this particular problem, but not in connection with Archytas or Menaechmus(!) What "counts" as μηχανικός is unfortuately even less clear. One possibility is a construction that has some element of *physical or imagined motion.* Change over time in a figure would seemingly violate Plato's vision of the eternal and unchanging nature of the world of the forms. Archytas' solution definitely qualifies as μηχανικός in that sense.

In conclusion, we can say that Plutarch has seemingly preserved a debate on mathematical methodology that is at least somewhat consistent with what we know from other sources. But from the work of Archytas and Menaechmus and then Archimedes, Apollonius and others, if something like Plato's criticism of the geometers in his circle actually happened, then its effect was minimal. Mathematics had emerged from philosophy as a subject in its own right, mechanics was entering its mainstream, and Plato's ideas about the proper methods or goals of geometry were hardly the last word.