Mathematics 304, section 2 – Ordinary Differential Equations
Problem Set 2
Due: Friday, September 17

A. From Section 1.8 in the handouts, do problem 27 – **Hint**: measure the amount of dioxin in units of "parts per billion times gallons."

B. From Chapter 1 in the main text, do problems 1, 2abe, 3, 4, 10ab.

C. (Differential equations with discontinuous coefficients). "Reasonable" solutions of first order linear differential equations $y' = g(t)y + r(t)$ can still be obtained in situations where one or both of $g(t), r(t)$ have jump discontinuities. In this problem, you will see how this works in the case

$$y' = \begin{cases} y - 4 & \text{if } t < 5 \\ 2 - y & \text{if } t \geq 5 \end{cases} \tag{1}$$

(that is,

$$g(t) = \begin{cases} 1 & \text{if } t < 5 \\ -1 & \text{if } t \geq 5 \end{cases} \qquad r(t) = \begin{cases} -4 & \text{if } t < 5 \\ 2 & \text{if } t \geq 5) \end{cases}$$

and various initial conditions $y(0) = y_0$. Here's the process: First, we find the general solution for $t < 5$ using the appropriate method. Determine the constant to satisfy the initial condition $y(0) = y_0$. Then find the general solution for $t \geq 5$ and choose the arbitrary constant there so that the resulting function is *continuous* at $t = 5$. The function obtained this way satisfies the equation (1), except at the point $t = 5$ (where the derivative is usually undefined).

1) Carry out this process for the initial condition $y(0) = 4$. Describe the qualitative behavior of this solution.
2) Same question for the initial condition $y(0) = 2$.
3) Give a general description of the qualitative behavior of solutions of (1), identifying all cases.

D. (More on solutions of the logistic equation) Recall that we have seen that solutions of $y' = ay(1 - y)$, $a > 0$ can be written in the form

$$y = \frac{Ke^{at}}{1 + Ke^{at}}$$

where K is an arbitrary constant, related to the value of y at time $t = 0$ by $K = \frac{y(0)}{1-y(0)}$.

1) Show that if $y(0) < 0$, then the solution becomes unbounded (approaches a vertical asymptote) in a finite amount of time.
2) Show that the solutions with $y(0) > 1$ also have vertical asymptotes if t is extended to negative values.