Mathematics 304 – Ordinary Differential Equations
Review Sheet – Final Examination
December 6, 2004

General Information

- The final examination for this class will be given during the scheduled period – 8:30 to 11:30 am on Thursday, December 16.
- The final will be a *comprehensive exam*, covering all the topics from the two midterms, *and the material about Hamiltonian systems and the undamped and damped pendulum systems from after the second midterm.* See the list of topics below for more details.
- The exam will be written to take about 2 hours if you work steadily, but you will have the full 3 hour period to use if you need that much time.
- The exam will be closed-book, calculators allows, and I will provide a table of integrals.
- If there is interest, I would be happy to arrange an evening review session during exam week (Monday or Tuesday would probably be the best days). We can discuss this in class on December 6.

How to Prepare

The best way to prepare for this exam will be to do (a lot of) specific problems related to this material. You will want to have the facility to do the calculations necessary without spending a lot of time deciding how to get started. We have done almost all of the relevant problems in the Hirsch, Smale, Devaney book, though (:( ) So, to study, I strongly suggest looking at problems from the Blanchard, Devaney, Hall book (on reserve in the Science Library). Chapters 1, 3, 4, 5 cover the same material that we have discussed (plus a few additional topics) and every section has lots of exercises!

Topics To Be Included

1) Terminology of ODE, solutions, initial value problems, the Existence and Uniqueness Theorem for solutions of first order initial value problems (one equation, one dependent variable).
2) Techniques for deriving analytic solutions of 1st order ODE: Separable equations, linear equations
3) Qualitative theory for 1st order ODE: Direction Fields, special techniques for autonomous equations (identifying equilibrium solutions, determining whether they are "sinks" or "sources" or neither)
4) 1st order ODE in modeling of mixing problems, heating/cooling, etc.
5) Bifurcation diagrams for 1-parameter families of ODE.
6) First order systems $X' = AX$ with constant coefficient matrix – solutions via eigenvalues, eigenvectors of the coefficient matrix. The canonical forms for higher-dimensional systems, and how to find solutions based on the canonical form.
7) The trace-determinant plane and the classification for 2×2 systems. Bifurcations in families.

8) Applications to unforced and forced oscillator equations:

$$mx'' + bx' + kx = g(t),$$

mass-spring systems, electrical circuits, etc. Resonance phenomena.

9) Non-linear first order systems – equilibrium points, the phase plane, nullclines, solutions

10) Linearization of a first order system at a critical point, classification of critical points in terms of eigenvalues of the linearized system.

11) Hamiltonian systems, their special properties, applications to the undamped and damped pendulum equations.

Suggested Review Problems

See review sheets for Midterm Exams 1 and 2 for topics 1-9 in the list above. For topics 10, 11, from Blanchard, Devaney, Hall:

Section 5.1/3, 11, 17
Section 5.2/1, 3
Section 5.3/3 (Hint: solve for x to sketch the level curves), 9, 11, 13,