Mathematics 304, section 2 – Ordinary Differential Equations
Review Sheet – Exam 1
September 24, 2004

General Information

The first midterm exam this semester will be given in class on Friday, October 1, as announced in the course syllabus. This will be a closed-book exam, but you may use calculators at any point, and I will provide a copy of a table of integrals for your use during the exam.

The exam will cover the material that we have studied since the start of the semester from the handout sections of Blanchard, Devaney and Hall ("BDH"), and Chapter 1 of Hirsch, Smale and Devaney ("HSD") (see specific topics and review problems below).

If there is interest, I would be happy to arrange a late afternoon or evening review session for the exam. Wednesday, September 29 is probably the best bet, although I will no doubt need to have a review session for my Principles of Analysis class that day too.

Topics To Be Included

1) ODE, solutions, initial value problems, the Existence and Uniqueness Theorem for solutions of first order initial value problems
2) Techniques for deriving analytic solutions of 1st order ODE: Separable equations, linear equations.
3) 1st order ODE in modeling: mixing problems, heating/cooling, etc.
4) Qualitative theory for 1st order ODE: Direction Fields, special techniques for autonomous equations (identifying equilibrium solutions, determining whether they are "sinks" or "sources"
5) Bifurcation diagrams for 1-parameter families of ODE.

Note: There are topics in Chapter 1 of HSD that we only just touched on (biggest example – the Poincaré map and its use in identifying periodic solutions of non-autonomous equations). The exam *will not* cover those topics.

Suggested Review Problems

A) From BDH, Section 1.2: 1, 7, 9, 11, 15, 25, 27, 31, 33, 35, 41

B) From BDH, Section 1.8/1, 3, 7, 9, 13, 21, 23 (this is like the mixing problems we did if you think about it the right way!) 29

C) Sketch the phase line for each equation, identify equilibria as sinks, sources, or neither. Then for the given initial conditions, sketch qualitative graphs of the corresponding solutions:

$$y' = y\cos y,\ y(0) = \pi, 0, 5\pi/3$$

$$y' = y^2 - 7y + 10,\ y(0) = 2, 3, 8, 1$$

Similar problems with $y' = f(y)$, but given a graph $z = f(y)$, not a formula (make up your own graphs to practice – include several maxima, minima, y-intercepts, etc.)

D) Sketch the phase line for the ODE

$$y' = \frac{1}{(y-2)(y+1)}$$

and discuss the behavior of the solution with $y(0) = 1/2$.

E) Sketch the bifurcation diagram for the family of equations

$$y' = y(1-y)^2 + a$$

and describe the bifurcations that occur as a increases from $-\infty$ to $+\infty$.

F) Same question for $y' = y^3 + ay - 1$.

G) Consider the population model $P' = 2P - P^2/50$ (t in years) for a species of fish in Crystal Lake (without fishing). Suppose fishing by humans will be allowed at Crystal Lake, and each licensee will catch 3 fish per year (these fish are hard to "land"!). How many licenses can be issued if the fish are to have a chance of surviving? (Hints: What differential equation models the population *with fishing*? If L is the number of licenses, how big can L be for your equation with fishing to still have real equilibria?) What will happen to the fish population if the maximum number of licenses is issued? How does the behavior depend on the initial population?