Mathematics 304, section 2 – Ordinary Differential Equations
Discussion 2 – Phase Portraits of 2×2 systems $X' = AX$
September 29, 2004

Goals

Today we want to work through a number of examples of 2×2 linear systems of ODE of the form

$$X'(t) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} X(t)$$

and understand the overall behavior of the set of solutions. The goal is to have each group work out one example and present your results to the whole class on the blackboard by the end of the class period. Then we can discuss the properties of all these examples.

Discussion Question

For your given system:

1) Determine the eigenvalues and eigenvectors for your matrix A.
2) Write down the general solution

$$X(t) = c_1 e^{\lambda_1 t} V_1 + c_2 e^{\lambda_2 t} V_2$$

3) Sketch the straight-line solutions (that is, solutions with $c_1 = 0$ or $c_2 = 0$, indicating the direction of increasing t. Then,
4) In a qualitative way, sketch the behavior of other solutions (that is, solutions where $X(0)$ is not an eigenvector), giving the "phase portrait" of the system.
5) Describe the overall behavior of solutions of your system.

Your Systems

Group 1 (Alison, Elizabeth C., Rebecca) – the system

$$X' = \begin{pmatrix} -3 & 2 \\ -1 & 0 \end{pmatrix} X$$

Group 2 (Nicole, Ben, Meghan) – the system

$$X' = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} X$$

Group 3 (Ingrid, Mackenzie, Trevor) – the system

$$X' = \begin{pmatrix} -8 & 10 \\ -5 & 7 \end{pmatrix} X$$

Group 4 (Molly, Bridget, Caitlin, Elizabeth V.) – the system

$$X' = \begin{pmatrix} 8 & -10 \\ 5 & -7 \end{pmatrix} X$$

Assignment

Each group is also responsible for a "clean copy" of your example, which will be distributed to the rest of the class as part of the notes for this day's class meeting. These are due on Monday, October 4.