MATH 241
Solution for Quiz 7
October 11, 2013

A) (20) Using the method of Lagrange Multipliers, find the maximum and minimum values of $f(x,y) = 2x + 3y$ on the constraint curve

$$E = \left\{ (x,y) \mid x^2 + \frac{y^2}{9} = 1 \right\}.$$

(If you run into trouble with the algebra, for more partial credit, sketch what the level curves of f should look like at the maximum and minimum together with the constraint curve.)

Solution: We have $f_x = 2$, $f_y = 3$ and $g_x = 2x$, $g_y = \frac{2y}{9}$. Therefore the Lagrange Equations are

$$\begin{aligned} 2 &= 2\lambda x \\ 3 &= \frac{2\lambda y}{9} \\ x^2 + \frac{y^2}{9} &= 1 \end{aligned}$$

If we multiply the first equation by y and the second by $27x$, then we get

$$2y = 2\lambda xy = 27x, \tag{1}$$

so $y = \frac{27x}{2}$. Substituting into the constraint equation, we get

$$x^2\left(1 + \frac{81}{4}\right) = 1 \Rightarrow x = \pm\frac{2}{\sqrt{85}}$$

Then equation (1) says

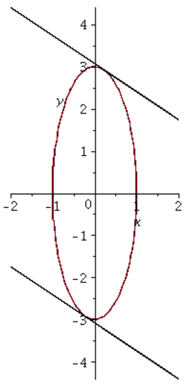
$$y = \pm\frac{27}{\sqrt{85}}$$

Note equation (1) implies that the positive sign for x gives a positive y and the negative sign for x gives a negative y. This means that there are *only two critical points of f restricted to E*:

$$\left(\frac{2}{\sqrt{85}}, \frac{27}{\sqrt{85}}\right), \left(\frac{-2}{\sqrt{85}}, \frac{-27}{\sqrt{85}}\right)$$

The function $f(x,y) = 2x + 3y$ has a positive value at the first point and a negative value at the second point, so the first gives the maximum value $\sqrt{85}$, and the second gives the minimum value $-\sqrt{85}$.

The geometric meaning of the Lagrange equations is that the level curves for $f(x,y) = 2x+3y$ at the maximum and minimum should be *tangent to the ellipse E*. The level curves of f are the lines $2x+3y = c$, or $y = \frac{-2}{3}x + \frac{c}{3}$, so these are straight lines of slope $m = \frac{-2}{3}$. Here is a picture of the constraint curve and the two contours for the maximum and minimum values:



4
3
y
2
1
-2
-1
0
1
2
x
-1
-2
-3
-4