6, 6, 6, 8, 4, 8, 24, 8   76

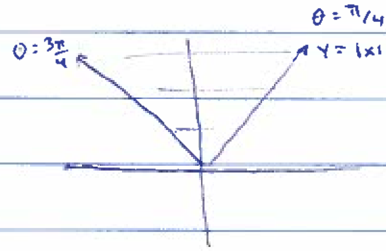
MATH 241 – PS 9 Solutions

§ 5.3 /

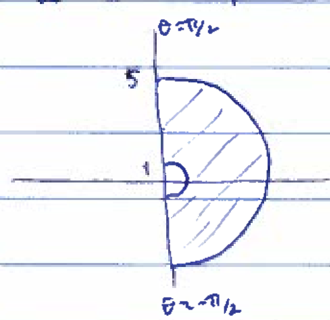
6   3. (a) $\{(r,\theta) \mid r \geq 0, \ \frac{\pi}{2} \leq \theta \leq \pi\}$ 2

(b) $\{(r,\theta) \mid r \geq 2, \ 0 \leq \theta < 2\pi\}$ 2

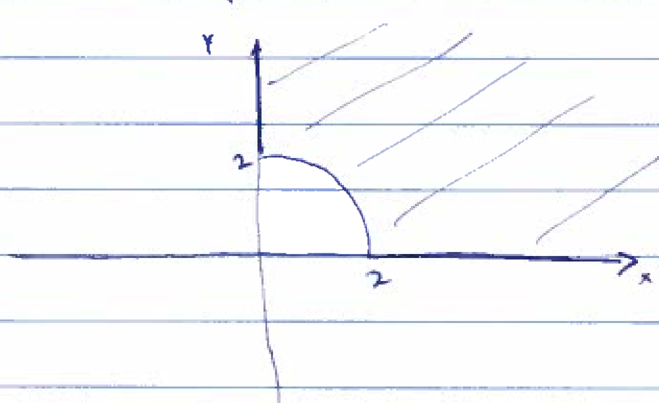
(c) $\{(r,\theta) \mid r \geq 0, \ \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}\}$ 2



6   4 (c)   the region bounded by the circles of radius 1, 5 and the lines x = 0 (y-axis) with x ≥ 0, including the circumferences   2



(d)   the region outside the circle $x^2+y^2=4$ in the 1st quadrant, including the arc of the circle and the portions of the x- and y-axes   2



2

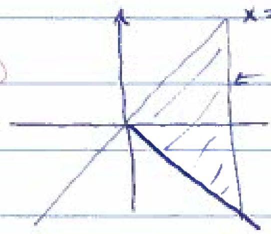
(e)   the union of the lines $y = \frac{x}{2}$ $\frac{x}{\sqrt{3}}$ and $y = -\frac{x}{2}$ $\frac{-x}{\sqrt{3}}$

(I'm assuming r is allowed to be negative too; if not then it's just the two rays)

6 5. (c) e^{-r^2} (2) (d) $\dfrac{r\sin\theta}{r\cos\theta} = \tan\theta$ (2) (e) $\dfrac{r\sin\theta}{r^2} = \dfrac{\sin\theta}{r}$ (2)

8 6. (a) (4) $\displaystyle\int_0^{\pi/2}\int_0^2 r\cos\theta \cdot r\,dr\,d\theta = \frac{r^3}{3}\Big|_0^2 \cdot \sin\theta\Big|_0^{\pi/2} = \boxed{\frac{8}{3}}$ 2 2

(d) (4)



$r\cos\theta = 2 \iff r = 2\sec\theta$

(also OK in rectangular coords: $\displaystyle\int_0^2\int_{-x}^{x} x^2-y^2\,dy\,dx$)

$$\int_{-\pi/4}^{\pi/4}\int_0^{2\sec\theta} x^2-y^2 \; r\,dr\,d\theta$$ 2

$$= \int_{-\pi/4}^{\pi/4}\int_0^{2\sec\theta} r^2(\cos^2\theta-\sin^2\theta)\cdot r\,dr\,d\theta$$

$$= \int_{-\pi/4}^{\pi/4}\int_0^{2\sec\theta} r^3\cdot(\cos^2\theta-\sin^2\theta)\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{r^4}{4}\Big|_0^{2\sec\theta}(\cos^2\theta-\sin^2\theta)\,d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 4\sec^4\theta\,(\cos^2\theta-\sin^2\theta)\,d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 4\sec^2\theta - 4\sec^2\theta\tan^2\theta\,d\theta$$

$$= 4\tan\theta - \frac{4}{3}\tan^3\theta\,\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= 8 - \frac{8}{3}$$

$$= \boxed{\frac{16}{3}}$$ ✓ 2

4 7. (a) (4) Put the center at $(0,0,0)$, take hemisphere with $z \geq 0$. Hemisphere is $z = \sqrt{4-r^2}$

$$V = \int_0^{2\pi}\int_0^2 \sqrt{4-r^2}\cdot r\,dr\,d\theta$$ 2

$$= \int_0^{2\pi} -\frac{1}{2}\cdot\frac{2}{3}(4-r^2)^{3/2}\Big|_{r=0}^{2}\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} 8 \, d\theta$$

$$= \boxed{\frac{16\pi}{3}}$$ 2

check:

$$\left(\frac{1}{2} \cdot \frac{4}{3}\pi (2)^3\right)$$

8 8. (b) polar:

$$\int_{\pi/2}^{3\pi/2}\int_0^3 (r\sin\theta - r\cos\theta)\, r \, dr\, d\theta$$ 2

4

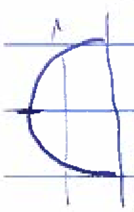
$$= \int_{\pi/2}^{3\pi/2} \frac{r^3}{3}\Big|_0^3 \cdot (\sin\theta - \cos\theta)\, d\theta$$

$$= 9\int_{\pi/2}^{3\pi/2} \sin\theta - \cos\theta \, d\theta$$

$$= 9\Big[-\cos\theta - \sin\theta\Big]_{\pi/2}^{3\pi/2}$$

$$= 9 + 9 = \boxed{18}$$ 2

4 rectangular:



$$\int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} y - x \, dy\, dx$$ 2

$$= \int_{-3}^{0} \frac{y^2}{2} - xy \Big|_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}} dx$$

$$= \int_{-3}^{0} (-2x)\sqrt{9-x^2}\, dx$$

$$= \frac{2}{3}(9-x^2)^{3/2}\Big|_{-3}^{0}$$

$$= \frac{2}{3}\cdot 27 = \boxed{18}.$$ 2

§ 5.4 /

(6) 1 (a) $\displaystyle\int_{-1}^{2}\int_{-1}^{0}\int_{0}^{1}(x^3y - z)\,dz\,dy\,dx$

$$= \int_{-1}^{2}\int_{-1}^{0} x^3yz - \frac{z^2}{2}\Big|_0^1 \,dy\,dx$$

$$= \int_{-1}^{2}\int_{-1}^{0} x^3y - \frac{1}{2}\,dy\,dx$$

$$= \int_{-1}^{2} \frac{x^3y^2}{2} - \frac{y}{2}\Big|_{-1}^{0}\,dx$$

$$= \int_{-1}^{2} -\frac{x^3}{2} - \frac{1}{2}\,dx$$

$$= -\frac{x^4}{8} - \frac{x}{2}\Big|_{-1}^{2}$$

$$= -2 - 1 + \frac{1}{8} - \frac{1}{2} = \frac{-16-8+1-4}{8} = \boxed{\frac{-27}{8}}\ \checkmark$$

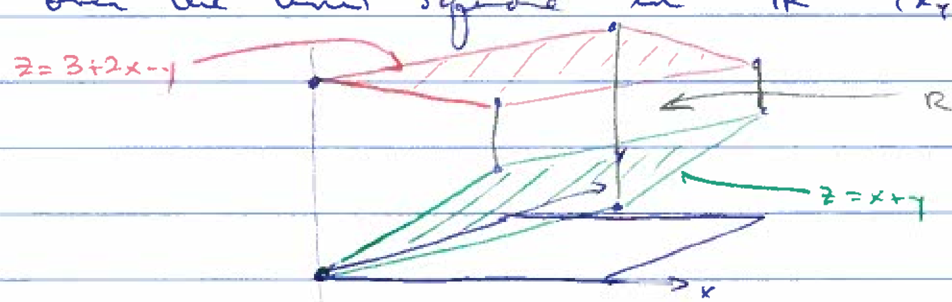
(6)

(24) 3 (a) $\displaystyle\int_{0}^{1}\int_{0}^{1}\int_{x+y}^{3+2x-y} xy\,dz\,dy\,dx$

label (8)

Region is volume between planes $z = x+y$ and $z = 3+2x-y$ over the unit square in $\mathbb{R}^2$ (x,y)

if vertical description [illegible]



$$= \int_0^1\int_0^1 xyz\Big|_{z=x+y}^{z=3+2x-y}\,dy\,dx$$

$$= \int_0^1\int_0^1 xy\left[(3+2x-y) - (x+y)\right]\,dy\,dx$$

$$= \int_0^1 \int_0^1 3xy + x^2y - 2xy^2 \, dy \, dx$$

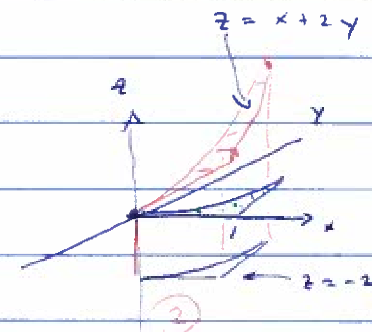
$$= \int_0^1 \frac{3xy^2}{2} + \frac{x^2y^2}{2} - \frac{2xy^3}{3} \Big|_0^1 \, dx$$

$$= \int_0^1 \frac{3x}{2} + \frac{x^2}{2} - \frac{2x}{3} \, dx$$

$$= \int_0^1 \frac{5x}{6} + \frac{x^2}{2} \, dx$$

$$= \frac{5x^2}{12} + \frac{x^3}{6} \Big|_0^1$$

$$= \boxed{\frac{7}{12}}$$ ✓ 6



again, just a written description is ok too

total 8 (c) $$\int_0^1 \int_0^{x^2} \int_{-2}^{x+2y} z \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{x^2} \frac{z^2}{2} \Big|_{-2}^{x+2y} \, dy \, dx$$

$$= \int_0^1 \int_0^{x^2} \frac{x^2}{2} + 2xy + 2y^2 - 2 \, dy \, dx$$

$$= \int_0^1 \frac{x^2y}{2} + xy^2 + \frac{2}{3}y^3 - 2y \Big|_0^{x^2} \, dx$$

$$= \int_0^1 \frac{x^4}{2} + x^5 + \frac{2x^6}{3} - 2x^2 \, dx$$

$$= \frac{x^5}{10} + \frac{x^6}{6} + \frac{2x^7}{21} - \frac{2x^3}{3} \Big|_0^1$$

$$= \frac{1}{10} + \frac{1}{6} + \frac{2}{21} - \frac{2}{3}$$

$2 \cdot 3 \cdot 5 \cdot 7 = 210$

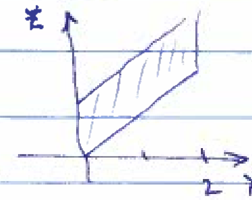
-7

$$= \frac{21 + 35 + 20 - 140}{210} = \boxed{\frac{-32}{105}}$$ 6 ✓

(e) $\int_0^2 \int_y^{y+1} \int_{z+y}^{z+y+1} zy\,dx\,dz\,dy$ region between parallel planes $x = z+y$, $x = z+y+1$

(8)

over



(2)

(a parallelogram)

$$= \int_0^2 \int_y^{y+1} xyz \Big|_{x=y+z}^{x=y+z-1} dz\,dy$$

$$= \int_0^2 \int_y^{y+1} yz\left[(y+z+1)-(y+z)\right] dz\,dy$$

$$= \int_0^2 \int_y^{y+1} yz\,dz\,dy$$

$$= \int_0^2 y \cdot \frac{z^2}{2} \Big|_y^{y+1} dy$$

$$= \int_0^2 y \cdot \left[\frac{(y+1)^2}{2} - \frac{y^2}{2}\right] dy$$

$$= \int_0^2 y\left(y + \tfrac{1}{2}\right) dy$$

$$= \frac{y^3}{3} + \frac{y^2}{4} \Big|_0^2$$

$$= \frac{8}{3} + 1 = \boxed{\frac{11}{3}}$$ ✓ 6

(8) 4 (c) Region between planes $z = y-1$, $z = x+1$ over rectangle $[-1,1] \times [0,1]$ in xy-plane

$$\int_{-1}^1 \int_0^1 \int_{y-1}^{x+1} xy\,dz\,dy\,dx$$

$$= \int_{-1}^1 \int_0^1 xyz \Big|_{y-1}^{x+1} dy\,dx$$

$$= \int_{-1}^1 \int_0^1 x^2y + xy - xy^2 + xy\,dy\,dx$$

$$= \int_{-1}^1 \int_0^1 x^2y + 2xy - xy^2\,dy\,dx$$

$$= \int_{-1}^1 \frac{x^2y^2}{2} + xy^2 - \frac{xy^3}{3} \Big|_0^1 dx$$

$$= \int_{-1}^1 \frac{x^2}{2} + x - \frac{x}{3}\,dy$$

(8)

$$= \frac{x^3}{6} + \frac{x^2}{3} \Big|_{-1}^1$$

$$= \boxed{\frac{1}{3}}$$ ✓