4, 4, 4, 8, 4, 4, 8, 6, 24 total is ~~76~~

MATH 241 - PS 8

§ 5.1 / 12, § 5.2 / 1a, 2b, 3cd, 4a, 7, 8a, 9a, 11ab

5.1/12 The best way to set up the partition of the room is to divide the $10' \times 12' \times 8'$ volume into 27 rectangular solids, each $\Delta V = \frac{10'}{3} \times 4' \times \frac{8'}{3} = \frac{320}{9}\ ft^3$ and place one of the measured points in each smaller solid. the average temperature is then

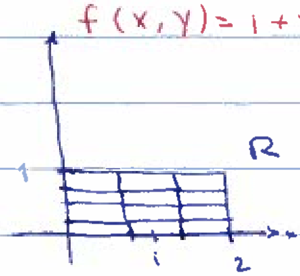
$$\frac{1}{960}\Big[71.4 + 72.3 + 72.1 + 69.3 + 69.4 + 71.3 + 62.9 + 58.3 + 64.9 + \cdots + 72.4\Big]\Delta V$$

$$= \frac{1}{27}\Big[\text{sum of the temp readings}\Big]$$

$$\doteq 70.72^\circ$$ (4)

§ 5.2/ $f(x,y) = 1 + x - y$

1(a)



$M = 3$, $N = 4$ $\Delta x = \frac{2}{3}$, $\Delta y = \frac{1}{4}$

$\Delta A = \Delta x \Delta y = \frac{1}{6}$

(4) Taking (x^*_{ij}, y^*_{ij}) = lower left corner in each rectangle

Note: any other choice of points is OK too - just check that process is OK

$$R(f, P, \delta) = 1 \cdot \tfrac{1}{6} + (1 + \tfrac{2}{3} - 0)\cdot\tfrac{1}{6} + (1 + \tfrac{4}{3} - 0)\cdot\tfrac{1}{6}$$
$$+ (1 - \tfrac{1}{4})\cdot\tfrac{1}{6} + (1 + \tfrac{2}{3} - \tfrac{1}{4})\cdot\tfrac{1}{6} + (1 + \tfrac{4}{3} - \tfrac{1}{4})\cdot\tfrac{1}{6}$$
$$+ (1 - \tfrac{1}{2})\cdot\tfrac{1}{6} + (1 + \tfrac{2}{3} - \tfrac{1}{2})\cdot\tfrac{1}{6} + (1 + \tfrac{4}{3} - \tfrac{1}{2})\cdot\tfrac{1}{6}$$
$$+ (1 - \tfrac{3}{4})\cdot\tfrac{1}{6} + (1 + \tfrac{2}{3} - \tfrac{3}{4})\cdot\tfrac{1}{6} + (1 + \tfrac{4}{3} - \tfrac{3}{4})\cdot\tfrac{1}{6}$$
$$= \boxed{31/12}$$ ✓

2 (b)

$$\iint_R (x-y)^3 \, dA = \int_0^1 \int_1^2 (x-y)^3 \, dy\, dx$$

$$= \int_0^1 -\frac{(x-y)^4}{4} \Big|_{1=y}^{2=y} dx$$

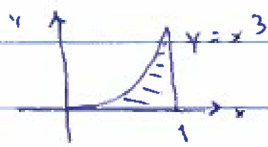
$$= \int_0^1 \frac{(x-1)^4}{4} - \frac{(x-2)^4}{4} \, dx$$ 2 partial credit

$$= \frac{(x-1)^5}{20}\Big|_0^1 - \frac{(x-2)^5}{20}\Big|_0^1$$

$$= \frac{1}{20} + \frac{1}{20} - \frac{32}{20}$$

$$= \boxed{-\frac{3}{2}}$$ 2 ✓ 4

3 (c) $\iint_R e^{y/x} \, dA$ $\quad R = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le x^3\}$



$$= \int_0^1 \int_0^{x^3} e^{y/x} \, dy\, dx$$

$$= \int_0^1 x e^{y/x} \Big|_{y=0}^{y=x^3} dx$$ 2

$$= \int_0^1 x e^{x^2} - x \, dx$$

$$= \frac{1}{2} e^{x^2}\Big|_0^1 - \frac{x^2}{2}\Big|_0^1$$

$$= \frac{1}{2}(e-1) - \frac{1}{2}$$

$$= \boxed{\frac{e}{2} - 1}$$ ✓

(d) $\iint_R y \, dA \qquad R = \{ (x,y) \mid 0 \le x \le \pi, \ 0 \le y \le \sin x \}$

$$= \int_0^{\pi} \int_0^{\sin(x)} y \, dy \, dx$$

$$= \int_0^{\pi} \frac{y^2}{2} \Big|_0^{\sin(x)} dx$$

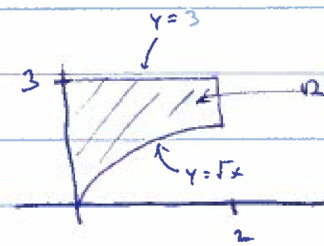
$$= \frac{1}{2} \int_0^{\pi} \sin^2 x \, dx$$

$$= \frac{1}{2} \int_0^{\pi} \frac{1}{2}(1 - \cos 2x) \, dx \qquad \text{(trig id.)}$$

$$= \frac{x}{4} - \frac{\sin(2x)}{8} \Big|_0^{\pi}$$

$$= \boxed{\frac{\pi}{4}}$$ ✓ 4

4 (a) $\int_0^2 \int_{\sqrt{x}}^3 xy \, dy \, dx$



$$= \int_0^2 \frac{xy^2}{2} \Big|_{\sqrt{x}}^3 dx$$

$$= \int_0^2 \frac{9}{2}x - \frac{x^2}{2} \, dx$$

$$= \frac{9x^2}{4} - \frac{x^3}{6} \Big|_0^2$$

$$= 9 - \frac{8}{6} = \frac{46}{6} = \boxed{\frac{23}{3}}$$ ✓

7. (a) Because $f(x,y)$ is continuous on R.

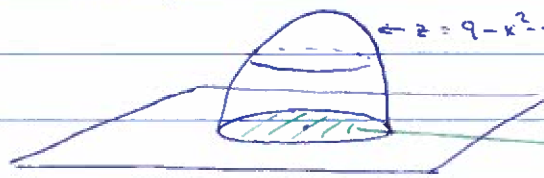
(b) By Fubini

$$\iint_R f(x,y)\, dA = \int_a^b \int_c^d g(x)h(y) \, dy \, dx = \int_a^b g(x) \cdot \left[\int_c^d h(y)\, dy \right] dx \quad \left(\begin{array}{l} \text{since} \\ g(x) \text{ is} \\ \text{indep of } y \end{array} \right)$$

$$= \int_a^b g(x)\,dx \cdot \int_c^d h(y)\,dy \qquad \left(\text{since } \int_c^d h(y)\,dy \text{ is constant}\right)$$

4

8(a) the surface $z = 9 - x^2 - y^2$ and the xy-plane ($z = 0$) intersect along the circle $x^2 + y^2 = 9$.



$R = \{(x,y) \mid x^2 + y^2 \le 9\}$
$= \{(x,y) \mid -3 \le x \le 3,\ -\sqrt{9-x^2} \le y \le +\sqrt{9-x^2}\}$

$$V = \iint_R 9 - x^2 - y^2 \, dA$$

$$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 9 - x^2 - y^2 \, dy\,dx \quad 2$$

$$= \int_{-3}^{3} \left. (9-x^2)\,y - \frac{y^3}{3} \right|_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}} dx$$

$$= \int_{-3}^{3} 2(9-x^2)^{3/2} - \frac{2}{3}(9-x^2)^{3/2} \, dx$$

$$= \frac{4}{3}\int_{-3}^{3} (9-x^2)^{3/2}\,dx \quad 2 \qquad \text{Let } x = 3\sin\theta, \quad dx = 3\cos\theta\,d\theta$$

$$= \frac{4}{3}\int_{-\pi/2}^{\pi/2} \cos^3\theta \cdot \cos\theta \, d\theta \cdot 81$$

8 total

$$= 108 \int_{-\pi/2}^{\pi/2} \cos^4\theta \, d\theta \quad 2$$

(Can do either with trig identities or with a table of integrals)

$$= \frac{108 \cdot 3\pi}{8}$$

$$= \boxed{\frac{81\pi}{2}} \quad 2$$

$= \left[\frac{1}{2}(1 + \cos 2\theta)\right]^2$
$= \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2 2\theta$
$= \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{8}(1 + \cos 4\theta)$
$= \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$

NB. It's also ok if they set this one up using polar coordinates:
$V = \int_0^{2\pi}\int_0^3 (9 - r^2)\,r\,dr\,d\theta$
$= \frac{81\pi}{2}$

9 (a)

$$\bar{f} = \frac{\int_0^1 \int_0^x x e^y \, dy \, dx}{\int_0^1 \int_0^x dy \, dx} \quad 2$$

$$= \frac{\int_0^1 x e^x - x \, dx}{\int_0^1 x \, dx} \quad 2 \qquad \leftarrow \text{use parts}$$

$$= \frac{x e^x - e^x - \frac{x^2}{2} \Big|_0^1}{\frac{x^2}{2} \Big|_0^1}$$

$$= \frac{e - e + 1 - \frac{1}{2}}{\frac{1}{2}}$$

$$= \boxed{1} \quad 2$$

11 (a)

$$m = \int_0^1 \int_{1-x}^1 c \, dy \, dx = \int_0^1 c x \, dx = c \frac{x^2}{2} \Big|_0^1 = \frac{c}{2} \quad 4$$

$$\int_0^1 \int_{1-x}^1 c x \, dy \, dx = c \int_0^1 x^2 \, dx = \boxed{\frac{c}{3}} \quad 4$$

$$\int_0^1 \int_{1-x}^1 c y \, dy \, dx = \int_0^1 \frac{c y^2}{2} \Big|_{1-x}^1 dx = \int_0^1 \frac{c}{2} - \frac{c}{2}(1-x)^2 \, dx$$

$$= c \int_0^1 \frac{1}{2} - \frac{1}{2} + x - \frac{x^2}{2} \, dx$$

$$= c \int_0^1 x - \frac{x^2}{2} \, dx$$

$$= c \left[\frac{x^2}{2} - \frac{x^3}{6} \right]_0^1 \quad 4$$

$$= \boxed{\frac{c}{3}}$$

So $\boxed{\bar{x} = \bar{y} = \frac{c/3}{c/2} = 2/3}$ (N.B. Expect by symmetry)

11 (b) $M = \int_0^1 \int_0^1 \cos x \, dy \, dx$

$= \int_0^1 \cos x \, dx$

$= \sin(1)$ 2

$\int_0^1 \int_0^1 x \cos x \, dy \, dx = \int_0^1 x \cos x \, dx$

$= x \sin x + \cos x \Big|_0^1$ (parts)

$= \sin(1) + \cos(1) - 1$ 2

$\int_0^1 \int_0^1 y \cos x \, dy \, dx = \int_0^1 \cos x \, dx \cdot \int_0^1 y \, dy$

$= \sin(1) \cdot \frac{1}{2}$ 2

So $\bar{x} = \dfrac{\sin(1) + \cos(1) - 1}{\sin(1)} \doteq .454$ ✓

$\bar{y} = \frac{1}{2}$ (symmetry ✓)