24, 18, 4, 4, 2, 8, 8, 4 72 total

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MATH 241 PS 5 Solutions (note strange order)

§ 3.2 /

10. (a) 6 $f(x,y) = x^2 y + y^3$

$\frac{\partial f}{\partial x} = 2xy$ $\frac{\partial^2 f}{\partial x^2} = 2y$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 2x$

$\frac{\partial f}{\partial y} = x^2 + 3y^2$ $\frac{\partial^2 f}{\partial y^2} = 6y$

1 point each

$-\frac{1}{2}$ for small slips

(OK if they ~~just give one~~ mixed partial)

(b) 6 $f(x,y) = e^{xy}$

$\frac{\partial f}{\partial x} = ye^{xy}$ $\frac{\partial^2 f}{\partial x^2} = y^2 e^{xy}$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = e^{xy} + xye^{xy}$

$\frac{\partial f}{\partial y} = xe^{xy}$ $\frac{\partial^2 f}{\partial y^2} = x^2 e^{xy}$

(c) 6 $f(x,y) = \sin(x)\cos(y)$

$\frac{\partial^2 f}{\partial x^2} = -\sin(x)\cos(y)$

$\frac{\partial f}{\partial x} = \cos(x)\cos(y)$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -\cos(x)\sin(y)$

$\frac{\partial f}{\partial y} = -\sin(x)\sin(y)$ $\frac{\partial^2 f}{\partial y^2} = -\sin(x)\cos(y)$

(d) 6 $f(x,y) = \frac{1}{x^2+y}$ $\frac{\partial^2 f}{\partial x^2} = \frac{6x^2 - 2y}{(x^2+y)^3}$ (simplified)

$\frac{\partial f}{\partial x} = \frac{-2x}{(x^2+y)^2}$ $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{4x}{(x^2+y)^3}$

$\frac{\partial f}{\partial y} = \frac{-1}{(x^2+y)^2}$ $\frac{\partial^2 f}{\partial y^2} = \frac{2}{(x^2+y)^3}$

12 (a) 9 $f(x,y,z) = xyz + x^2$

$f_x = yz + 2x$ $f_y = xz$ $f_z = xy$

$f_{xx} = 2$, $f_{xy} = f_{yx} = z$, $f_{xz} = f_{zx} = y$, $f_{yz} = f_{zy} = x$

$f_{yy} = 0$, $f_{zz} = 0$.

(b) $f_x = \dfrac{x}{\sqrt{x^2+y^2+z^2}}$, $f_y = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$, $f_z = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$f_{xx} = \dfrac{y^2+z^2}{(x^2+y^2+z^2)^{3/2}}$, $f_{yy} = \dfrac{x^2+z^2}{(x^2+y^2+z^2)^{3/2}}$, $f_{zz} = \dfrac{x^2+y^2}{(x^2+y^2+z^2)^{3/2}}$

9

$f_{xy} = f_{yx} = \dfrac{-xy}{(x^2+y^2+z^2)^{3/2}}$, $f_{xz} = f_{zx} = \dfrac{-xz}{(x^2+y^2+z^2)^{3/2}}$,

$f_{yz} = f_{zy} = \dfrac{-yz}{(x^2+y^2+z^2)^{3/2}}$. (Note symmetry!)

15. $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

(a) if $(x,y) \neq (0,0)$

1 $\dfrac{\partial f}{\partial x} = \dfrac{(x^2+y^2)(y) - xy \cdot 2x}{(x^2+y^2)^2} = \dfrac{y^3 - x^2 y}{(x^2+y^2)^2}$

1 $\dfrac{\partial f}{\partial y} = \dfrac{(x^2+y^2)(x) - xy \cdot 2y}{(x^2+y^2)^2} = \dfrac{x^3 - xy^2}{(x^2+y^2)^2}$

b) if $(x,y) = (0,0)$,

1 $\dfrac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \dfrac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0} \dfrac{0-0}{h} = 0$

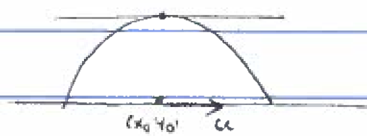
1 $\dfrac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \dfrac{f(0,h) - f(0,0)}{h} = \lim_{h \to 0} \dfrac{0-0}{h} = 0$

16. $P = \dfrac{nRT}{V}$ (note $n, R, T, V > 0$)

(a) $\dfrac{\partial P}{\partial T} = \dfrac{nR}{V}$ 1 and $\dfrac{\partial P}{\partial V} = \dfrac{-nRT}{V^2}$ 1

(b) A unit change in T increases P (since $\frac{\partial P}{\partial T} > 0$)
A unit change in V decreases P (since $\frac{\partial P}{\partial V} < 0$) 2

5. $D_u f(x_0, y_0) = 0$ for all unit vectors u if (x_0, y_0) is the location of Mount Blue. Every slice by a vertical plane has a local max. at (x_0, y_0)



so $D_u f(x_0, y_0) = 0$ 2

6. (a) $f(x,y) = xy$ so $\frac{\partial f}{\partial x} = y$, $\frac{\partial f}{\partial y} = x$ 2

at $(x,y) = (x_0, y_0)$, $D_{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})} f(x_0, y_0) = (y_0, x_0) \cdot (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$$= \boxed{\frac{y_0}{\sqrt{2}} + \frac{x_0}{\sqrt{2}}}$$ 2

$$D_{(0,1)} f(x_0, y_0) = (y_0, x_0) \cdot (0,1)$$
$$= \boxed{x_0}$$

(c) $f(x,y) = x^2 + y^3$, so $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 3y^2$ 2

$$D_{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})} f(x_0, y_0) = (2x_0, 3y_0^2) \cdot (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$= \sqrt{2}\, x_0 + \frac{3y_0^2}{\sqrt{2}} = \boxed{\sqrt{2}\, x_0 + \frac{3\sqrt{2}\, y_0^2}{2}}$$ 2

$$D_{(0,1)} f(x_0, y_0) = (2x_0, 3y_0^2) \cdot (0,1)$$
$$= \boxed{3y_0^2}$$

§ 3.4 /

5 (b) $f(x,y) = \cos(x+y) - \sin(x-y)$ at $(\frac{\pi}{2}, 0)$

$$f(\tfrac{\pi}{2}, 0) = \cos(\tfrac{\pi}{2}) - \sin(\tfrac{\pi}{2}) = -1$$

$$\frac{\partial f}{\partial x} = -\sin(x+y) - \cos(x-y)$$

so $\frac{\partial f}{\partial x}(\frac{\pi}{2}, 0) = -\sin(\frac{\pi}{2}) - \cos(\frac{\pi}{2}) = -1$

$$\frac{\partial f}{\partial y} = -\sin(x+y) + \cos(x-y)$$

so $\frac{\partial f}{\partial y}(\frac{\pi}{2}, 0) = -\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2}) = -1$

tangent plane is $z = -1 - (x - \frac{\pi}{2}) - y$

$$\boxed{z = \left(\frac{\pi}{2} - 1\right) - x - y}$$ 4

(d) $f(x,y) = \frac{1}{x^2+y^2}$ at $(1,2)$

$f(1,2) = \frac{1}{5}$

$\frac{\partial f}{\partial x} = \frac{-2x}{(x^2+y^2)^2}$, so $\frac{\partial f}{\partial x}(1,2) = \frac{-2}{25}$

$\frac{\partial f}{\partial y} = \frac{-2y}{(x^2+y^2)^2}$, so $\frac{\partial f}{\partial y}(1,2) = \frac{-4}{25}$

tangent plane is $z = \frac{1}{5} - \frac{2}{25}(x-1) - \frac{4}{25}(y-2)$

or $\boxed{z = -\frac{2}{25}x - \frac{4}{25}y + \frac{3}{5}}$ 4

§ 3.5 /

1 (a) Direct method:

$(f \circ \alpha)(t) = t^2 - t(4t)^3 = t^2 - 64t^4$

so $\frac{d}{dt}(f \circ \alpha)(t) = 2t - 256t^3$ 2

By Chain Rule: $\alpha'(t) = (1,4)$

$\nabla f(x,y) = (2x - y^3, -3xy^2)$, so

$\nabla f(\alpha(t)) \cdot \alpha'(t) = (2t - 64t^3, -48t^3) \cdot (1,4)$

$= 2t - 64t^3 - 192t^3$

$= 2t - 256t^3$ 2 Same!