54 total

241 PS4 : 2.3/ 13, 14 ; 2.4/ 2b, 3, 4d, 9, 3.1/ 9, 10

2 2 4 10 4 5c 8 use maple 6 16 2

2.3/13 (a) the numbers of susceptible and infected individuals now oscillate, but with decreasing amplitude, tending to values about S = 200 and I = 50

(b) the immigration could be modeled by adding additional terms to the right side(s) of the SIR equations. Assuming the immigration happens at a constant rate (# people per time unit) and none of the new people are infected or recovered, the model might change to

2

$$\begin{cases} \dfrac{dS}{dt} = -rSI + i \\ \dfrac{dI}{dt} = rSI - aI \\ \dfrac{dR}{dt} = aI \end{cases}$$

(i = constant immigration term)

(Note if some of the immigrants were infected or recovered, we might want to add constant terms to all the equations:

$$\begin{cases} \dfrac{dS}{dt} = -rSI + i_S \\ \dfrac{dI}{dt} = rSI - aI + i_I \\ \dfrac{dR}{dt} = aI + i_R \end{cases}$$

also ok if they do this

14. Vaccination would remove individuals from the susceptible group and place them into the recovered (immune group):

2 $\dfrac{dS}{dt} = -rSI - v$; $\dfrac{dI}{dt} = rSI - aI$; $\dfrac{dR}{dt} = aI + v$

2.4 /

2 (b) $F(x,y) = (x-1, y)$

(i) $\alpha(t) = (2e^t+1, 3e^t)$ satisfies $\alpha'(t) = (2e^t, 3e^t)$

and $F(\alpha(t)) = ((2e^t+1)-1, 3e^t) = (2e^t, 3e^t)$. Since

$\alpha'(t) = F(\alpha(t))$, this is a flowline. 2 (3,3)

(ii) $\alpha(t) = (1-e^t, -e^t)$ satisfies $\alpha'(t) = (-e^t, -e^t)$

and $F(\alpha(t)) = ((1-e^t)-1, -e^t) = (-e^t, -e^t)$. So

$\alpha'(t) = F(\alpha(t))$. 2 one for each side of equation (0,-1)

3. $F(x,y) = (2x, y)$

(a) the flow line ~~equations~~ are

$x' = 2x \Rightarrow x(t) = x(0)e^{2t}$

$y' = y \Rightarrow y(t) = y(0)e^t$

initial point $(-2,1)$: $(-2e^{2t}, e^t)$ 6

$(-1,3)$: $(-e^{2t}, 3e^t)$

$(0,1)$: $(0, e^t)$

(b) $(-2,1)$: $x = -2y^2$

$(-1,3)$ $x = -(\frac{1}{3}y)^2 = -\frac{1}{9}y^2$ 2

$(0,1)$ $x = 0$

(c) this will be true only if $y(0) \neq 0$. If

2 $\alpha(0) = (1,0)$, for instance we get $\alpha(t) = (e^{2t}, 0)$

which gives a curve along the x-axis $(y=0)$, not of

the form $x = g(y)$.

4 (d). $\cancel{\alpha(t) = (Ae^t + Bte^t, (A+B)e^{-t} + Bte^t)}$

~~satisfies~~ $\cancel{\alpha'(t) = (-Ae^t + Be^t + Bte^t, -(A+B)e^{-t} + Be^t + Bte^t)}$

$\alpha'(t) = (Ae^t(\cos t + \sin t) + Ae^t(-\sin t + \cos t) + Be^t(\sin t - \cos t) + Be^t(\cos t + \sin$
$- Ae^t \sin t + Be^t \cos t - Ae^t \cos t - Be^t \sin t)$

$= (2Ae^t \cos t + 2Be^t \sin t, \underbrace{-Ae^t(\sin t + \cos t) + Be^t(\cos t - \sin t)}_{= -x \text{ here } \alpha})$

$F(\alpha(t)) = (2Ae^t(\cos t + \sin t) + 2Be^t(\sin t - \cos t) \cancel{- 2Ae^t \sin t + 2Be^t \cos t},$
$-[Ae^t(\sin t + \cos t) + Be^t(\sin t - \cos t)])$

$= (2Ae^t \cos t + 2Be^t \sin t, -[Ae^t(\sin t + \cos t) + Be^t(\sin t - \cos t)]$

(4) Since $\alpha' = F(\alpha)$, this gives flow lines.

(i) To get $\alpha(0) = (1,0)$ we want $(A - B, B) = (1,0)$,
so $B = 0$ and $A = 1$: $\alpha(t) = (e^t(\cos t + \sin t), -e^t \sin t)$

(ii) To get $\alpha(0) = (-1,1)$, $(A - B, B) = (-1, 1)$ so
$B = 1$, $A = 0$: $\alpha(t) = (e^t(\sin t - \cos t), e^t \cos t)$

5 (c) $\begin{cases} \cos(x-y) = 0 \\ xy = 0 \end{cases}$ define the critical points

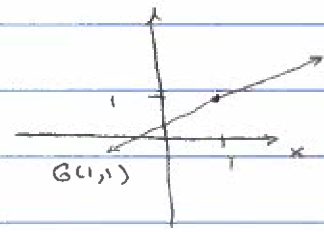
Second equation gives $x = 0$ or $y = 0$
If $x = 0$ $\cos y = 0$ so $y = \pm\frac{\pi}{2} \doteq \pm 1.57$ in given range
(8) If $y = 0$ $\cos x = 0$ so $x = \pm\frac{\pi}{2}$

So critical points are $(\frac{\pi}{2}, 0)$ — saddle
$(-\frac{\pi}{2}, 0)$ — saddle
$(0, \frac{\pi}{2})$ — source (Spiral type)
$(0, -\frac{\pi}{2})$ — sink (Spiral type)

9 (a) the plot of G should show all vectors multiplied by -1. For instance $F(1,1) = (2,1)$

so $G(1,1) = (-2,-1)$

2



(b) If $\alpha(t)$ is a flow line of F, so $x'(t) = 2x(t)$ and $y'(t) = y(t)$, then $\beta(t) = (x(-t), y(-t))$ will be a flow line of G: $\beta'(t) = (-x'(-t), -y'(-t))$

2

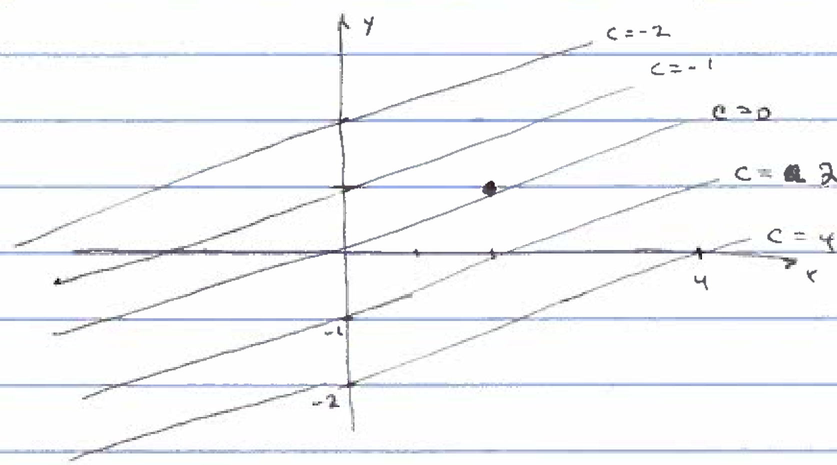
$= (-2x(-t), -y(-t)) = G(\beta(t))$

(c) F has a critical point at $(0,0)$ that is a <u>source</u>; G has a critical point at $(0,0)$ too, but it is a <u>sink</u>. 2

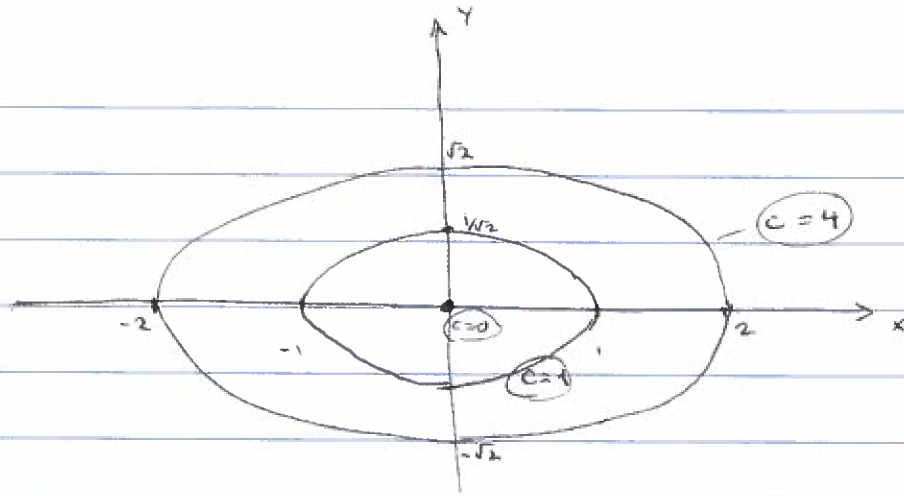
4 each: must show at least 3 contours (c values) for full credit

3.1 / 9

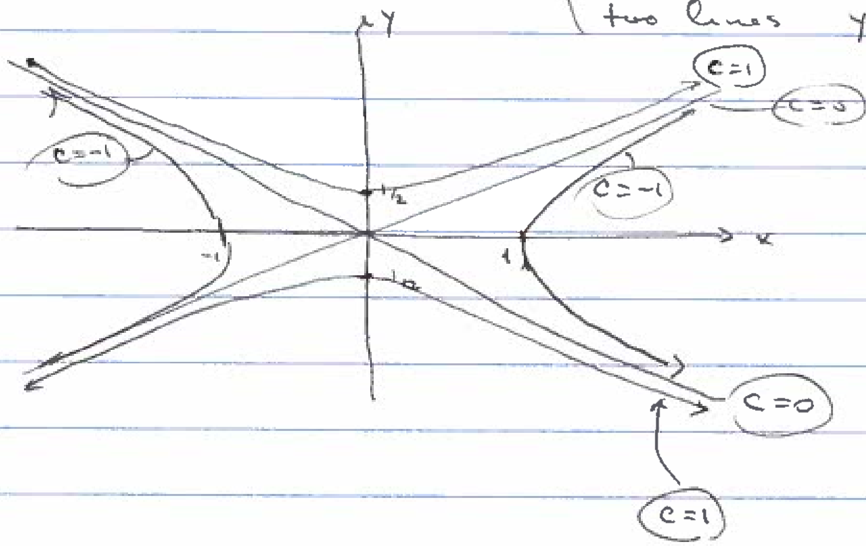
(a) $x - 2y = c$ is the <u>straight line</u> $y = \frac{1}{2}x - \frac{c}{2}$



(b) $x^2 + 2y^2 = c$ is $\begin{cases} \text{an } \underline{\text{ellipse}} \text{ if } c > 0 \\ \text{a point if } c = 0 \\ \text{empty if } c < 0 \end{cases}$

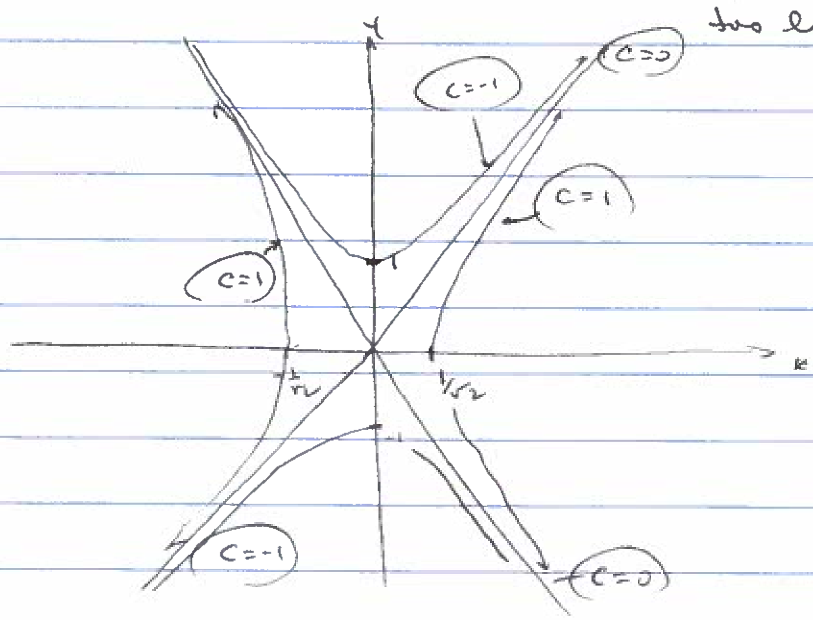


(c) $-x^2+4y^2=c$ is $\begin{cases}\text{a hyperbola if } c\neq 0\\ \text{two lines } y=\pm\frac{1}{2}x \text{ if } c=0\end{cases}$



(d) $2x^2-y^2=c$ is a hyperbola if $c\neq 0$

two lines $y=\pm\sqrt{2}\,x$ if $c=0$



10. the countour plot (a) shows that there is some sort of maximum or minimum around $(x, y) = (1, 0)$. But since ~~[illegible]~~ contour values are not given, we cannot tell which type of point that is. the surface plot (b) lets us see that the point is a maximum ("peak").

2