Solutions for PS 1 : 1.1/ 1cd, 4abc, 7, 10ab, 12, 13
1.2/ 2, 4, 5, 10, 14, 15

§ 1.1/ 1 (c) the top edge of the quadrilateral is the line $y = -\frac{1}{2}x + \frac{3}{2}$ and the bottom edge is the line $y = -x - 1$. So the region is

$$R = \{(x,y) \in \mathbb{R}^2 \mid -1 \le x \le 1, \quad -x-1 \le y \le -\tfrac{1}{2}x + \tfrac{3}{2}\}$$

(d) the region lies inside the circle of radius 1, center at $(1,0)$, and to the ~~upper~~ right of $x = 1$. So

$$R = \{(x,y) \in \mathbb{R}^2 \mid (x-1)^2 + y^2 \le 1, \quad x \geq 1\}$$

(other possible forms come by solving for x or y in equation of the circle)

$$= \{(x,y) \in \mathbb{R}^2 \mid 1 \le x \le 1 + \sqrt{1-y^2}, \quad -1 \le y \le 1\}$$

$$= \{(x,y) \in \mathbb{R}^2 \mid -\sqrt{1-(x-1)^2} \le y \le \sqrt{1-(x-1)^2}, \quad 1 \le x \le 2\}$$

4 (a) the plane parallel to the yz-plane through $(\pi, 0, 0)$

(b) the line parallel to the z-axis through $(5, 6, 0)$ (intersection of the planes $x=5$, $y=6$.)

(c) $(y-3)(z-3) = 0$ if either $y = 3$ or $z = 3$ (or both). So the set defined here is the union of two planes: one parallel to xz-plane, through $(0,3,0)$; one parallel to xy-plane through $(0,0,3)$.

7. Place the center of the Earth at the origin. The radius is about 3959 miles. So the region is

$$\{ (x,y,z) \in \mathbb{R}^3 \mid (3959)^2 < x^2 + y^2 + z^2 \leq (4159)^2 \}$$

(assumes satellite is not on the ground)

10(a) Planes $z = c$ give $x^2 + y^2 = 1 - c^2/4$

so the slices are <u>circles</u> if $-2 < c < 2$

<u>points</u> if $c = \pm 2$

<u>empty</u> if $c < -2$ or $c > 2$

Planes $y = b$ give $x^2 + \frac{z^2}{4} = 1 - b^2$,

so the slices are <u>ellipses</u> if $-1 < b < 1$

<u>points</u> if $b = \pm 1$

<u>empty</u> if $b > 1$ or $b < -1$

Planes $x = a$ give $y^2 + \frac{z^2}{4} = 1 - a^2$,

so the slices are <u>ellipses</u> if $-1 < a < 1$

<u>points</u> if $a = \pm 1$

<u>empty</u> if $a > 1$ or $a < -1$

(this is an <u>ellipsoid</u>)

(b) Planes $z = c$ give $x^2 + y^2 = 1 + \frac{c^2}{4}$

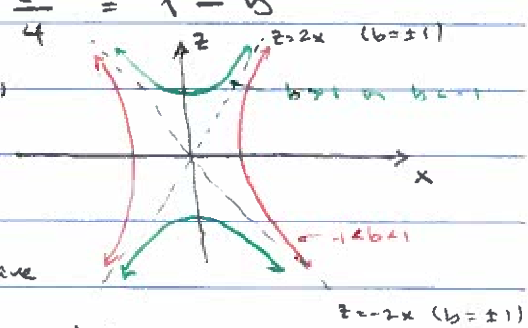
slices are <u>circles</u> for all c. (the radius increases as $|c|$ increases.)

Planes $y = b$ give $x^2 - \frac{z^2}{4} = 1 - b^2$

slices are <u>hyperbolas</u> if $b \neq \pm 1$,

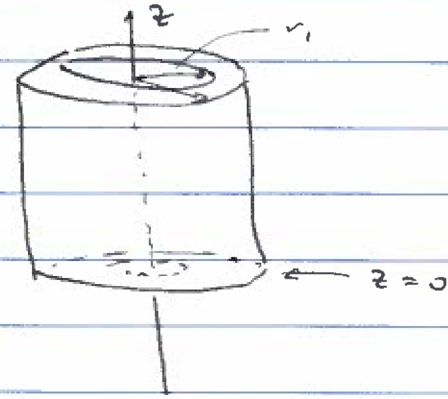
<u>unions of two lines</u> if $b = \pm 1$

$(z = \pm 2x)$



Planes $x = a$ are similar; slices are hyperbolas if $a \neq \pm 1$, two lines if $a = \pm 1$.

12\. (a) Place the z-axis through the center of the heart cavity, and $z=0$ at bottom:



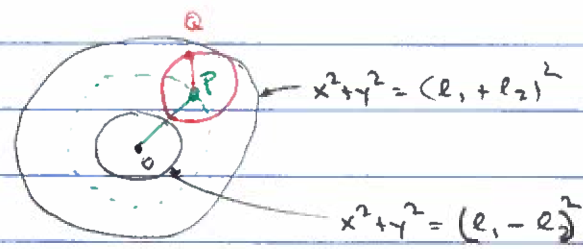
then the region is

$$\{(x,y,z)\in\mathbb{R}^3 \mid r_1^2 \le x^2+y^2 \le r_2^2,\ 0\le z\le h\}$$

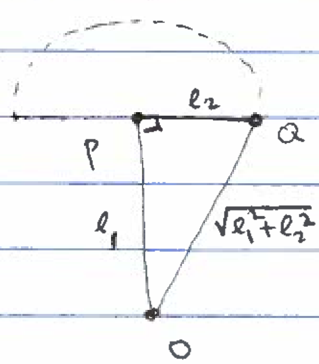
(b) We want $\begin{cases} \pi r_1^2 h = 40 \\ \pi r_2^2 h - \pi r_1^2 h = 100 \end{cases}$

there are infinitely many solutions. taking $h = 5$ cm, for instance, gives $\pi r_1^2 = 8$ so $r_1 = \sqrt{8/\pi} \doteq 1.6$ cm and $r_2 = \sqrt{28/\pi} \doteq 3$ cm

13\. (a) Q can be placed at any point in the annular region: $\{(x,y)\in\mathbb{R}^2 \mid (\ell_1-\ell_2)^2 \le x^2+y^2 \le (\ell_1+\ell_2)^2\}$



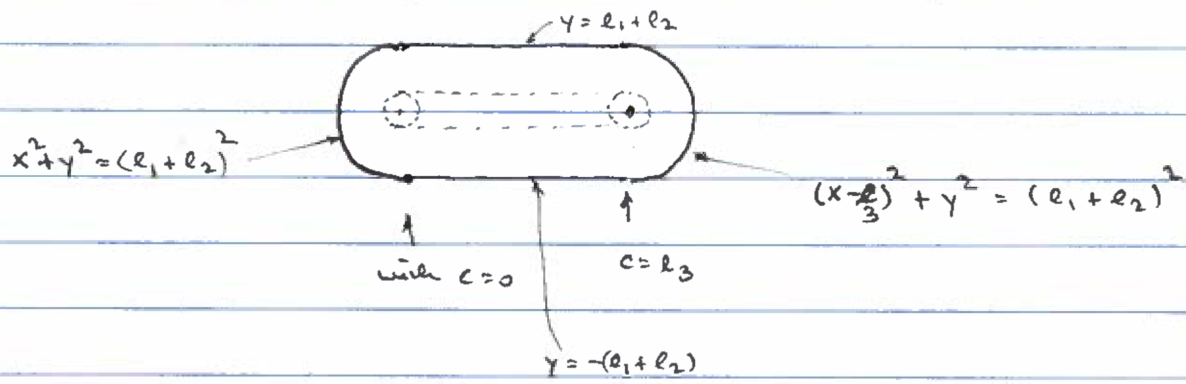
(b) the closest Q can be to the origin now is when $OP \perp PQ$: the region is a smaller annulus



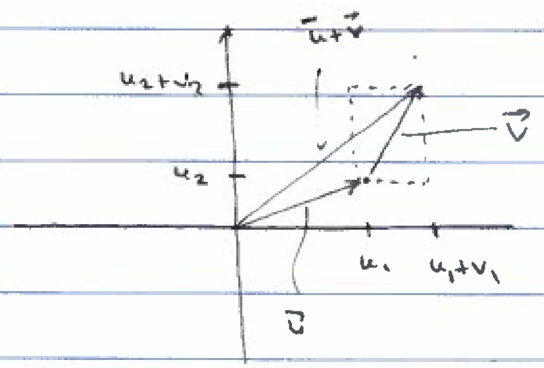
Q can be any point in $\{(x,y)\in\mathbb{R}^2 \mid \ell_1^2+\ell_2^2 \le x^2+y^2 \le (\ell_1+\ell_2)^2\}$

(c) For convenience, say $A = (0,0)$
$B = (\ell_3, 0)$
so $O = (c, 0)$ for some $0 \le c \le \ell_3$. By letting O slide along the segment from A to B, we see that the set of possible positions of Q is a union of annuli as in part (a) (but with center at $(c,0)$), one for each c, $0 \le c \le \ell_3$. Since $\ell_3 > 2(\ell_1 - \ell_2)$ ($2(\ell_1-\ell_2)$ is the diameter of the inner circle in each annulus), the "holes" are completely filled by the other annuli, and the set of possible positions of Q is

$$\left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; -\sqrt{(\ell_1+\ell_2)^2 - y^2} \le x \le \ell_3 + \sqrt{(\ell_1+\ell_2)^2-y^2}, \quad -(\ell_1+\ell_2) \le y \le \ell_1+\ell_2 \right\}$$

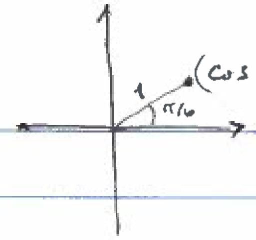


§1.2 2.



Place the tail of $\vec{u}$ at $(0,0)$, the coordinates of the head are then at (u_1,u_2). If the tail of $\vec{v}$ is placed at the head of $\vec{u}$, then the head of $\vec{v}$ is at (u_1+v_1, u_2+v_2)

4. (a) (i)

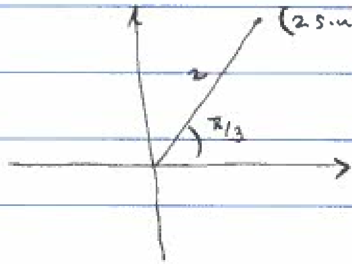


$(\cos(\pi/6), \sin(\pi/6)) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\|\vec{v}\| = \sqrt{\cos^2\frac{\pi}{6} + \sin^2\frac{\pi}{6}} = 1$

(ii)

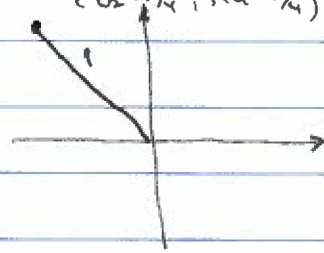
$(2\sin\frac{\pi}{6}, 2\cos\frac{\pi}{6}) = (1, \sqrt{3})$



$\|\vec{v}\| = \sqrt{1^2 + 3^2} = 2$

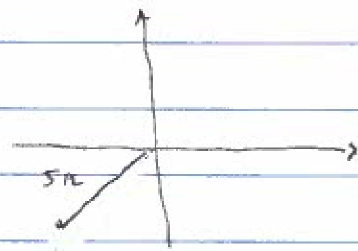
(iii)

$(\cos 3\pi/4, \sin 3\pi/4) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$



$\|\vec{v}\| = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

(iv)

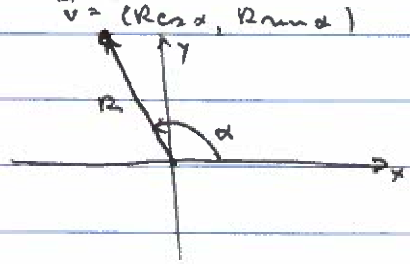


$\|v\| = \sqrt{(\frac{5}{2})^2\cdot\frac{1}{2} + (\frac{5}{2})^2\cdot\frac{1}{2}} = \frac{5}{2}$

$(\frac{5}{2}\sin(-\frac{3\pi}{4}), \frac{5}{2}\cos(-\frac{3\pi}{4})) = (-\frac{5}{2}, -\frac{5}{2})$

$\vec{v} = (R\cos\alpha, R\sin\alpha)$

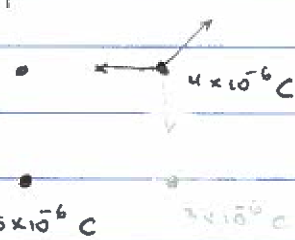
(b)



$\|\vec{v}\| = \sqrt{R^2\cos^2\alpha + R^2\sin^2\alpha} = |R|$

makes angle α with positive x-axis.

10 (a)



Force on 3rd from 1st is $\dfrac{(4\times10^{-6})(5\times10^{-6})}{4\pi\varepsilon_0(\sqrt{2})^3}(1,1)$

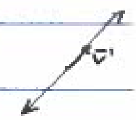
$= \dfrac{\cancel{2}\times10^{-11}}{4\pi\varepsilon_0\cdot\cancel{2}\sqrt{2}}(1,1)$

Force on 3rd from 2nd is $\dfrac{(4\times10^{-6}(-3\times10^{-6})}{4\pi\varepsilon_0\ (1)^3}(1,0)$

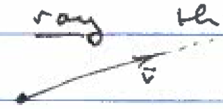
(b) Total force on 3rd is the vector sum:

$$\frac{10^{-12}}{4\pi\varepsilon_0}\left(5\sqrt{12}-12,\ 5\sqrt{12}\right)$$

14. (a) the line through (0,0,0) in direction of $\vec{v}$

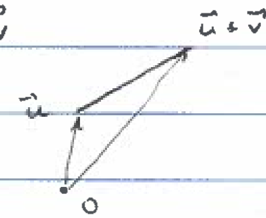


(b) the ray through (0,0,0) " " "



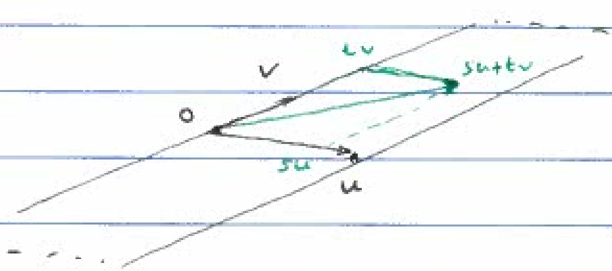
(c) the line segment from (0,0,0) to the head of $\vec{v}$

(d) the line segment from $\vec{u}$ to $\vec{u}+\vec{v}$



15. (b) (Assuming $\vec{u}, \vec{v}$ not collinear) this is the parallelogram:

(a) (Assuming $\vec{u}, \vec{v}$ not collinear), an infinite strip



(c) As in (a), but whole strip is shifted, so it passes through W: