$\underbrace{4}_{5.1} + \underbrace{16 + 8 + 8 + 8 + 6 + 6}_{5.2} + \underbrace{6 + 6 + 8 + 4 + 8}_{5.3}$ Grand Total 88

MATH 241 PS 8

§5.1 / 5. Subdivide the 10×10 square with $M = N = 4$

So there are 16 smaller squares each $2.5 \times 2.5 = 6.25 \text{ cm}^2$

4 →

Using the given heights, we have a Riemann sum

$$V \doteq (1+2+2+1 + 2+5+5+2 + 2+5+5+2 + 1+2+2+1)\cdot 6.25 \text{ cm}^3$$

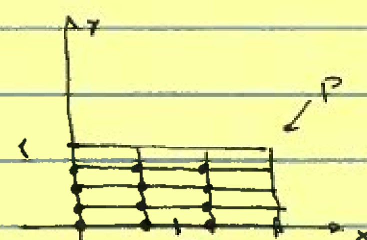
$$= 40 \times 6.25$$

$$= 250 \text{ cm}^3.$$

5.2/ 1(a) $f(x,y) = 1 + x - y$

$R = [0,2] \times [0,1]$

$M = 3,\ N = 4$



2

total 8 { Can use $\Delta x = \frac{2}{3}$, $\Delta y = \frac{1}{4}$ 2 (other selections OK

Selecting the lower left corner point in each small rectangle to evaluate $f(x,y)$,

4 (in form of sum (actual value is not unique)

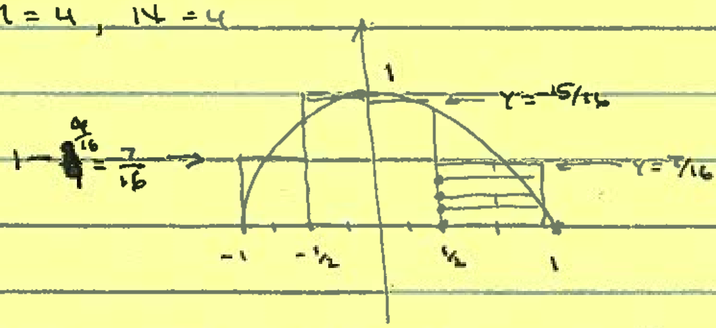
$$R(f,P) = \sum_{i=1}^{3}\sum_{j=1}^{4} f(x_{ij}^*, y_{ij}^*)\,\Delta y\,\Delta x$$

$$= \left(1 + \frac{5}{3} + \frac{7}{3} + \frac{3}{4} + \frac{17}{12} + \frac{25}{12} + \frac{1}{2} + \frac{7}{6} + \frac{11}{6} + \frac{1}{4} + \frac{11}{12} + \right.$$ $\cdot\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)$ —

$= \boxed{\dfrac{31}{12}}$ (other selections of points possible too.)

(b) $f(x,y) = x^2 - y^2$ $\quad R = \{(x,y) \mid -1 \le x \le 1,\ 0 \le y \le 1-x^2\}$

$M = 4,\ N = 4$



8

$\Delta x = \frac{1}{2}$, for $-1 \le x \le -\frac{1}{2}$ use $\Delta y = \frac{7}{16} \cdot \frac{1}{4} = \frac{7}{64}$

$-\frac{1}{2} \le x \le 0$ use $\Delta y = \frac{15}{16} \cdot \frac{1}{4} = \frac{15}{64}$

$0 \le x \le \frac{1}{2}$ use $\Delta y = \frac{15}{16} \cdot \frac{1}{4} = \frac{15}{64}$

$\frac{1}{2} \le x \le 1$ use $\Delta y = \frac{7}{64}$

(x_{ij}^*, y_{ij}^*) = lower left corner of R_{ij}

$$R(f,P) = \left(\left(\left(\tfrac{1}{2}\right)^2 - 0^2\right) + \left(\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{7}{64}\right)^2\right) + \left(\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{14}{64}\right)^2\right) + \left(\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{21}{64}\right)^2\right)\right) \cdot \left(\tfrac{1}{2}\right.$$

$$+ \left(\left((0)^2 - (0)^2\right) + \left((0)^2 - \left(\tfrac{15}{64}\right)^2\right) + \left((0)^2 - \left(\tfrac{30}{64}\right)^2\right) + \left((0)^2 - \left(\tfrac{45}{64}\right)^2\right)\right) \cdot \left(\tfrac{1}{2}\right) \cdot ($$

$$+ \left(\left(\left(-\tfrac{1}{2}\right)^2 - (0)^2\right) + \left(\left(-\tfrac{1}{2}\right)^2 - \left(\tfrac{15}{64}\right)^2\right) + \left(\left(-\tfrac{1}{2}\right)^2 - \left(\tfrac{30}{64}\right)^2\right) + \left(\left(-\tfrac{1}{2}\right)^2 - \left(\tfrac{45}{64}\right)^2\right)\right)($$

$$+ \left(\left((-1)^2 - (0)^2\right) + \left((-1)^2 - \left(\tfrac{7}{64}\right)^2\right) + \left((-1)^2 - \left(\tfrac{14}{64}\right)^2\right) + \left((-1)^2 - \left(\tfrac{21}{64}\right)^2\right)\right)\left(\left(\tfrac{1}{2}\right)\right.$$

(Again, lots of different ways of subdividing, selections of points are possible!)

[Just check for correct set up of Riemann sum — don't grade for final numerical value.]

2. (a) $\iint_R x^3 y^2 \, dA = \int_{-1}^{1}\int_0^5 x^3 y^2 \, dy \, dx$

$= \int_{-1}^{1} \frac{x^3 y^3}{3} \Big|_{y=0}^{y=5} dx$

$= \int_{-1}^{1} \frac{125 x^3}{3} \, dx$

$= \frac{125 x^4}{12} \Big|_{-1}^{1} = \boxed{0}$ 4

(d) $\iint_R xy e^{x+y} \, dA = \int_0^2 \int_{-2}^{0} x y e^{x+y} \, dy \, dx$

$= \int_0^2 x \left[y e^{x+y} - e^{x+y} \right] \Big|_{y=-2}^{y=0} dx$ (parts)

$= \int_0^2 x \left(-e^x + 2e^{x-2} + e^{x-2} \right) dx$

$= \int_0^2 -xe^x + 3xe^{x-2} \, dx$

$= -xe^x + e^x + 3xe^{x-2} - 3e^{x-2} \Big|_0^2$ (parts)

$= -2e^2 + e^2 + 6 - 3 - 0 - 1 - 0 + 3e^{-2}$

$= \boxed{-e^2 + 2 + 3e^{-2}}$ 4 ✓

3 (a) $\iint_R -x + y \, dA = \int_0^1 \int_{x^4}^{x} y - x \, dy \, dx$

$= \int_0^1 \frac{y^2}{2} - xy \Big|_{y=x^4}^{y=x} dx$

$= \int_0^1 \frac{x^2}{2} - x^2 - \frac{x^8}{2} + x^5 \, dx$

$= \int_0^1 -\frac{x^2}{2} - \frac{x^8}{2} + x^5 \, dx$

3 (a) cont

$$= -\frac{x^3}{6} - \frac{x^9}{18} + \frac{x^6}{6} \Big|_0^1$$

$$= -\cancel{\frac{1}{6}} - \frac{1}{18} + \cancel{\frac{1}{6}} - 0$$

$$= \boxed{-\frac{1}{18}}$$

4

(c) $$\iint_R e^{y/x} \, dA = \int_0^1 \int_0^{x^3} e^{y/x} \, dy \, dx$$

$$= \int_0^1 x e^{y/x} \Big|_{y=0}^{y=x^3} dx$$

$$= \int_0^1 x e^{x^2} - x \, dx$$

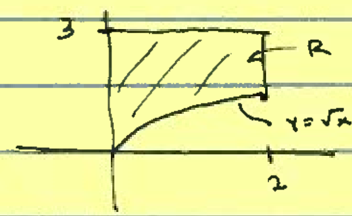
$$= \frac{1}{2} e^{x^2} - \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1}{2} e - \frac{1}{2} - \frac{1}{2} + 0$$

$$= \boxed{\frac{1}{2} e - 1}$$

4

4. (a) $$\int_0^2 \int_{\sqrt{x}}^3 xy \, dy \, dx$$



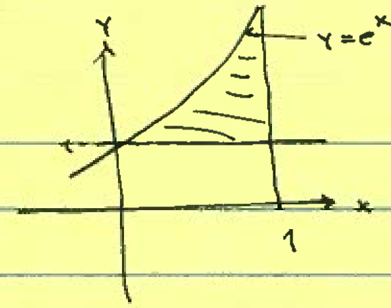
$$= \int_0^2 \frac{xy^2}{2} \Big|_{\sqrt{x}}^3 dx$$

$$= \int_0^2 \frac{9x}{2} - \frac{x^2}{2} \, dx$$

4

$$= \frac{9x^2}{4} - \frac{x^3}{6} \Big|_0^2 = 9 - \frac{8}{6} = \boxed{\frac{23}{3}}$$

(b) $\int_0^1 \int_1^{e^x} x + y \; dy \, dx$



$$= \int_0^1 xy + \frac{y^2}{2} \Big|_1^{e^x} dx$$

$$= \int_0^1 x e^x + \frac{e^{2x}}{2} - x - \frac{1}{2} \, dx$$

$$= x e^x - e^x + \frac{1}{4} e^{2x} - \frac{x^2}{2} - \frac{x}{2} \Big|_0^1$$

$$= \cancel{e} - \cancel{e} + \frac{1}{4} e^2 - \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{2}} - 0 + \cancel{1} - \frac{1}{4} + 0$$

$$= \boxed{\frac{1}{4}(e^2 - 1)} \quad \checkmark$$

4

8 (b) $z = x^2 + 2y^2 \qquad z = 12 - 2x^2 - y^2$

meet where $x^2 + 2y^2 = 12 - 2x^2 - y^2$

or $\boxed{x^2 + y^2 = 4}$

$$V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 12 - 3(x^2 + y^2) \, dy \, dx$$

3 for setup

$$= \int_{-2}^{2} 12y - 3x^2 y - y^3 \Big|_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} dx$$

$$= \int_{-2}^{2} 24\sqrt{4-x^2} - 6x^2\sqrt{4-x^2} - 2(4-x^2)\sqrt{4-x^2} \, dx$$

$$= \int_{-2}^{2} 16\sqrt{4-x^2} - 4x^2\sqrt{4-x^2} \, dx$$

table

$$= 16\cdot\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]\Bigg|_{-2}^{2}$$

$$- 4\left[\frac{x}{8}(2x^2-4)\sqrt{4-x^2} + \frac{16}{8}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 16\left[0 + \pi - 0 + \pi\right] - 4\left[\pi + \pi\right]$$

3) Cr value

$$= 32\pi - 8\pi = \boxed{24\pi}$$

check with polar:

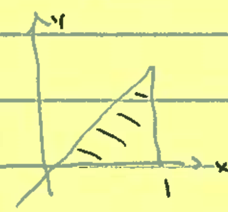
$$= \int_0^{2\pi}\int_0^2 (12-3r^2)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} 6r^2 - \frac{3r^4}{4}\Bigg|_0^2 d\theta$$

$$= (24 - 12)\cdot 2\pi$$

$$= 24\pi \checkmark$$

9 (a) $f(x,y) = xe^y$ $\quad R = \{(x,y) \mid 0\le x\le 1,\ 0\le y\le x\}$



$\int_0^1 xe^x - x\,dx$

average value

$$\bar{z} = \frac{\int_0^1\int_0^x xe^y\,dy\,dx}{\int_0^1\int_0^x 1\,dy\,dx} = \frac{\int_0^1 xe^y\Big|_0^x dx}{\int_0^1 x\,dx} = \frac{xe^x - e^x - \frac{x^2}{2}\Big|_0^1}{\frac{x^2}{2}\Big|_0^1}$$

6

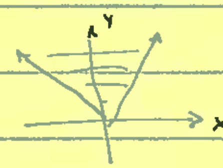
$$= \frac{e - e + 1/2}{\frac{1}{2}} = \boxed{1}$$

§ 5.3

3 (a) $\{(r,\theta) \mid \frac{\pi}{2}\le\theta\le\pi\}$ 2

(and $r\ge 0$)

(c) $\{(r,\theta) \mid 2\le r\le 4\}$ 2

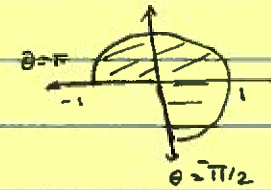


(e) $\{(r,\theta) \mid \frac{\pi}{4}\le\theta\le\frac{3\pi}{4}\}$ 2

(and $r\ge 0$)

4) (b) $\{(r,\theta) \mid r \le 1, \ -\frac{\pi}{2} \le \theta \le \pi\}$

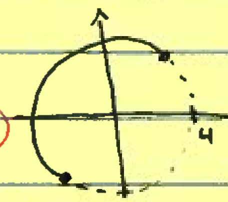
the portion of the disk of radius 1 center (0,0) in quadrants 1, 2, 4



2

(d) $\{(r,\theta) \mid r = 4, \ \frac{\pi}{4} \le \theta \le \frac{5\pi}{4}\}$

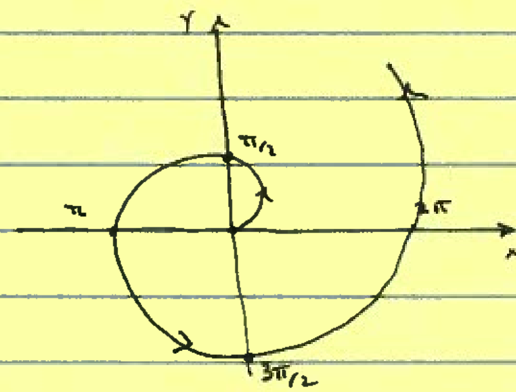
the semicircle of the circle of radius 4, center (0,0) extending from $\theta = \frac{\pi}{4}$ to $\theta = \frac{5\pi}{4}$



2

(not disk, just part of boundary circle)

(f) $\{(r,\theta) \mid r \ge 0, \ \theta \ge 0\}$: a spiral (of Archimedes)



2

6 (a) $$\int_0^{2\pi}\int_0^1 (1-r^2)\, r\, dr\, d\theta = \int_0^{2\pi} \frac{r^2}{2} - \frac{r^4}{4}\Big|_0^1 d\theta$$

$$= \frac{1}{4}\cdot 2\pi = \boxed{\frac{\pi}{2}}$$ ✓ 4

(b) $$\int_0^{\pi/2}\int_0^2 r\cos\theta \cdot r\, dr\, d\theta = \frac{r^3}{3}\Big|_0^2 \cdot \sin\theta\Big|_0^{\pi/2}$$

$$= \boxed{\frac{8}{3}}$$ ✓ 4

$x^2+y^2+z^2=4 \Rightarrow z=\sqrt{4-x^2-y^2}$

7 (a) $$V=\int_0^{2\pi}\int_0^2 \sqrt{4-r^2}\cdot r\,dr\,d\theta$$

$$=\int_0^{2\pi} -\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}(4-r^2)^{3/2}\Big|_0^2\,d\theta$$

$$=\int_0^{2\pi}\frac{1}{3}4^{3/2}\,d\theta$$ 4

$$=\int_0^{2\pi}\frac{8}{3}\,d\theta = \boxed{\frac{16\pi}{3}}$$

check $\frac{4\pi(2^3)}{3}\cdot\frac{1}{2}$ ✓

8 (a)

rect: $$\int_{-2}^{2}\int_0^{\sqrt{4-x^2}} y^2\,dy\,dx$$

$$=\int_{-2}^{2}\frac{1}{3}(4-x^2)^{3/2}\,dx$$ 4

table $$=\frac{1}{3}\left[-\frac{x}{8}(2x^2-20)\sqrt{4-x^2}+\frac{48}{8}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{+2}$$

$$=\frac{1}{3}\left[6\cdot\frac{\pi}{2}+6\frac{\pi}{2}\right]$$

$$=\boxed{2\pi}$$

polar: $$\int_0^{\pi}\int_0^2 r^2\sin^2\theta\; r\,dr\,d\theta$$ 4

$$=\int_0^{\pi}\frac{r^4}{4}\cdot\sin^2\theta\Big|_0^2\,d\theta$$

$$=\int_0^{\pi}4\sin^2\theta\,d\theta = 4\cdot\left[\frac{\theta}{2}-\frac{1}{4}\sin 2\theta\right]\Big|_0^{\pi}=\boxed{2\pi}$$