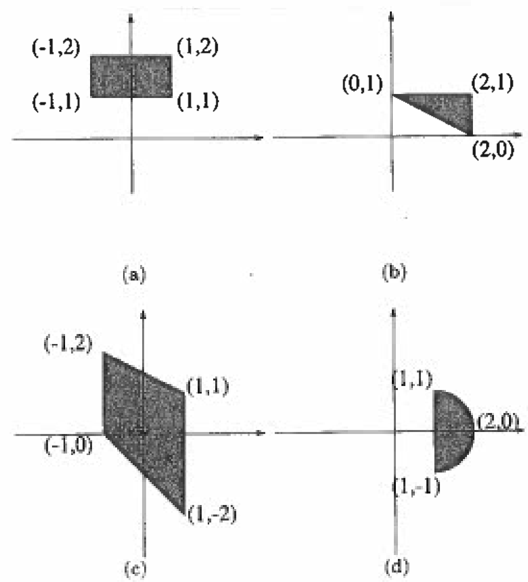
Mathematics 241, section 1 – Multivariable Calculus
Discussion 1 – The Cartesian Coordinate System in $\mathbf{R}^3$
September 7, 2010

Background

Over the last couple of days, we introduced Cartesian coordinates in $\mathbf{R}^2$ and $\mathbf{R}^3$ and we saw how various geometric objects in the plane or space could be described using these coordinates. Today, in our first small-group discussion day, we want to practice some more using these ideas. Each group should write up *one set of solutions to these questions to hand in.* If you are done by the end of the period, you can hand them in at that time; if not, arrange to meet outside of class to finish up. The write-ups are due no later than the start of class on Wednesday, September 8.

I. Use set notation to give a symbolic description of each of the subsets of $\mathbf{R}^2$ shown below:



II. Describe each of the following subsets of $\mathbf{R}^3$ in words. If the set is defined by more than one equation or inequality, say what you get with each one separately, and how that relates to the set you are looking at.

A) $S_1 = \{(x, y, z) \in \mathbf{R}^3 : x = 3 \text{ and } z = 3\}$

B) $S_2 = \{(x, y, z) \in \mathbf{R}^3 : x^2 + y^2 - 6y + z^2 = 0\}$ (complete the square in y)

III. We will study the set $C = \{(x, y, z) \in \mathbf{R}^3 : x^2 + y^2 - z^2 = 0\}$ by the "method of slices" in this question. (Note: This looks similar to an example we saw in class, but the right-hand side of the equation is *zero*, not *one*. This makes a difference!)

A) What are the slices of C by planes $z = c$? How is the case $c = 0$ different from the others?

B) What are the slices of of C by planes $x = c$? Again, the case $c = 0$ is different, what is the slice in that case?

C) Draw a reasonably accurate picture of C. What is the usual name of this surface?

D) The equation $z = 2x + 1$ defines a slanted plane in $\mathbf{R}^3$ passing through the point $(0, 0, 1)$ on the z-axis. What happens if you substitute for z in the equation of C – what kind of curve in the xy-plane is defined by the equation you get?

E) The equation $z = y + 1$ defines another slanted plane in $\mathbf{R}^3$ passing through the point $(0, 0, 1)$ on the z-axis. What happens if you substitute for z in the equation of C – what kind of curve in the xy-plane is defined by the equation you get?

F) Why are circles, ellipses, hyperbolas, and parabolas called *conic sections*?

IV. In class we introduced the vector sum $v + w$ in geometrical and coordinate form. There is also a scalar multiplication operation $c \cdot v$ which is defined like this: If $v = (v_1, v_2)$ (respectively (v_1, v_2, v_3)), then $c \cdot (v_1, v_2) = (cv_1, cv_2)$ (resp. (cv_1, cv_2, cv_3)).

A) Draw $v = (1, 2)$ together with $\frac{1}{2} \cdot (1, 2), 2 \cdot (1, 2), (-1) \cdot (1, 2)$ in the plane. On the basis of this, how are $c \cdot v$ and v related in general? (Say how the magnitude and direction of $c \cdot v$ relate to the magnitude and direction of v.)

B) Now draw the vectors

$$(-1, 3) + (1, 2), (-1, 3) + \frac{1}{2} \cdot (1, 2), (-1, 3) + 2 \cdot (1, 2), (-1, 3) + (-1) \cdot (1, 2)$$

as arrows, all with tail at the origin in $\mathbf{R}^2$.

C) If you formed the vectors $(-1, 3) + t \cdot (1, 2)$ for all $t \in \mathbf{R}$, what kind of geometric figure would be formed by the set of heads of these vectors?