A) (5) Sketch the level curves with $c = -1, 0, 1$ for the function $f(x, y) = x^2 - y^2$.

Solution: The level curve for the value $c$ is $x^2 - y^2 = c$. When $c = 0$, we get the two lines $y = \pm x$. With $c \neq 0$, the level curve is a hyperbola. The hyperbola opens left and right if $c > 0$. The hyperbola opens up and down if $c < 0$. Here is a plot, from Maple:

![Hyperbola Plot](image)

B) (5) Let

$$f(x, y) = \begin{cases} \frac{x + y}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Does $f$ have a limit as $(x, y) \to (0, 0)$ along the line $y = 0$? (Be careful, $+\sqrt{a^2} = |a|$, not $a$.) What does your answer say about the existence of $\lim_{(x,y) \to (0,0)} f(x, y)$?

Solution. If $y = 0$, then $f(x, 0) = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$, which is $+1$ if $x > 0$ and $-1$ if $x < 0$. This says,

$$\lim_{x \to 0^+} f(x, 0) = -1 \neq 1 = \lim_{x \to 0^-} f(x, 0).$$

Since there is not even a single limit along this one line, the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ cannot exist.