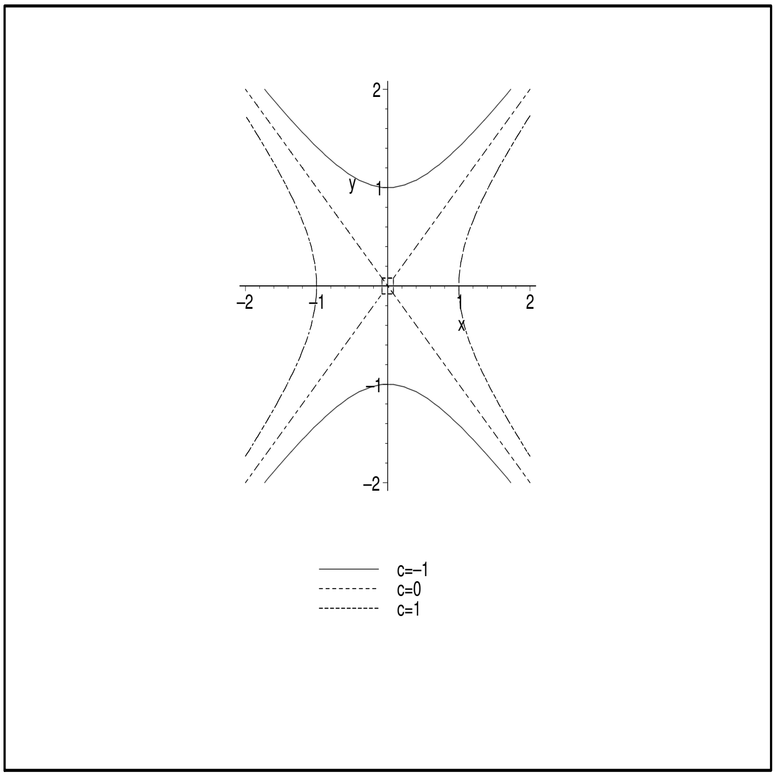
Mathematics 241 – Multivariable Calculus
Quiz 3 Solutions – September 21, 2007

A) (5) Sketch the level curves with $c = -1, 0, 1$ for the function $f(x, y) = x^2 - y^2$.

Solution: The level curve for the value c is $x^2 - y^2 = c$. When $c = 0$, we get the two lines $y = \pm x$. With $c \neq 0$, the level curve is a hyperbola. The hyperbola opens left and right if $c > 0$. The hyperbola opens up and down if $c < 0$. Here is a plot, from Maple:



B) (5) Let

$$f(x, y) = \begin{cases} \frac{x+y}{\sqrt{x^2+y^2}} & \text{if}(x, y) \neq (0, 0) \\ 0 & \text{if}(x, y) = (0, 0) \end{cases}$$

Does f have a limit as $(x, y) \to (0, 0)$ along the line $y = 0$? (Be careful, $+\sqrt{a^2} = |a|$, not a.) What does your answer say about the existence of $\lim_{(x,y)\to(0,0)} f(x, y)$?

Solution. If $y = 0$, then $f(x, 0) = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$, which is $+1$ if $x > 0$ and -1 if $x < 0$. This says,

$$\lim_{x\to 0^-} f(x, 0) = -1 \neq 1 = \lim_{x\to 0^+} f(x, 0).$$

Since there is not even a single limit along this one line, the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ cannot exist.