MONT 104N – Modeling the Environment
Solutions for Sample Midterm Exam Problems
October 22, 2012

Sample Mathematical Questions

A. (Refer to the tables on pages 6 - 8; something like those would be supplied for your use on a question like this.)

1. A committee on rail freight transport recommends increasing the mass that can be carried in a trailer from 42 to 46 tonnes. How much are these masses in kilograms? In pounds?

   Solution: One tonne is 1000 kilograms. So the recommendation is that the mass increases from 42000 kg to 46000 kg. Similarly, from the table on page 8, we see 1 lb = .4536 kg, so one 1 kg = 1/.4536 lb = 2.2046 lb. This gives the increase as from 92593.2 lb to 101411.6 lb.

2. About 24 million liters of rainwater are discharged by a storm drain into a river. The concentration of toxic arsenic in the rainwater averages 170 micrograms of arsenic per liter. How many kilograms of arsenic are discharged into the river each year.

   Solution: Since one microgram is 10^{-6} gram, the amount of arsenic discharged is

$$170 \times 10^{-6} \frac{\text{g}}{\text{liter}} \times 24 \times 10^{6} \text{ liter} \times 10^{-3} \frac{\text{kg}}{\text{g}} = 4.08 \text{ kg}.$$

B. In 2002, 10731 species were listed on the World Conservation Union "red list" of endangered species. In 2004, the corresponding number was 15042.

1. What was the ratio of the size of the 2004 list to the size of the 2002 list?

   Solution: The ratio is $\frac{15042}{10731} \doteq 1.402$.

2. What was the percentage increase in the size of the list from 2002 to 2004?

   Solution: The percentage increase is

$$\frac{15042 - 10731}{10731} \times 100\% = 40.2\%$$

   (Note that you can "see this" in the answer to part 1 if you subtract the leading 1.)

3. It is not known exactly how many species of plants and animals there are on the Earth today. But say that number is estimated as 12 million. What percentage of that total number appeared on the red list in each year?

Solution: In 2002, the percent was

$$\frac{10731}{12000000} \times 100\% \doteq .089\%$$

Similarly, in 2004, the percent was

$$\frac{15042}{12000000} \times 100\% \doteq .125\%$$

(around 1 tenth of one percent in each case). These are not huge numbers of course, but any loss of biodiversity means that potentially valuable or useful biochemical products of these plants and animals would be lost.

C. The following table gives several African countries, their total forest areas, and their areas of legally protected forests (in square kilometers)

Country	Total	Protected
Cameroon	289965	17845
Congo	278979	12935
Gabon	239369	8975
Zaire	1439178	93160

1. Which country has the largest protected forest area?

   Solution: The numerically largest protected forest area is in Zaire.

2. Which country has the largest percentage of protected forest?

   Solution: The percentages of protected forest are

$$\begin{aligned} \text{Cameroon}: \quad & \frac{17845}{289965} \times 100\% = 6.15\% \\ \text{Congo}: \quad & \frac{12935}{278979} \times 100\% = 4.64\% \\ \text{Gabon}: \quad & \frac{8975}{239369} \times 100\% = 3.75\% \\ \text{Zaire}: \quad & \frac{93160}{1439178} \times 100\% = 6.47\% \end{aligned}$$

   So Zaire also has the largest percentage of protected forest.

D. A cubic meter of air has a mass of about 1.25 kg. About 2 mg of that mass is methane. What is the mass concentration of methane in air, expressed in parts per million?

Solution: The mass concentration can be computed like this, converting the numerator and denominator to units of kilograms:

$$\frac{2 \times 10^{-6}}{1.25} = 1.6 \times 10^{-6} = \frac{1.6}{10^6},$$

so: 1.6 parts per million methane. (Note: Methane is a much more "potent" greenhouse gas than carbon dioxide. It has 25 times the impact of carbon dioxide in trapping heat from solar radiation in the atmosphere.)

E. The US EPA Toxic Release Inventory for 2000 includes the following information:

State	Toxics (in 10^6lb)
Nevada	1000
Utah	956
Arizona	744
Alaska	535
Texas	302

1. Construct a bar chart showing this information. Label your vertical axis.

   Solution: Mark the vertical axis in units of $100 \times 10^6 = 10^8$ lb of toxics released. Then the bar chart will have 1 bar of height 10 for Nevada, one of height 9.56 for Utah, etc.

2. Construct a pie chart showing the portions of the total toxics from these five states that each state contributes.

   Solution: The total is 3537×10^6 lb of toxics released. So the portions are:

$$\begin{aligned} \text{Nevada}: &\ \frac{1000}{3537} \times 100\% \doteq 28\% \\ \text{Utah}: &\ \frac{956}{3537} \times 100\% \doteq 27\% \\ \text{Arizona}: &\ \frac{744}{3537} \times 100\% \doteq 21\% \\ \text{Alaska}: &\ \frac{535}{3537} \times 100\% \doteq 15\% \\ \text{Texas}: &\ \frac{302}{3537} \times 100\% \doteq 8.5\% \end{aligned}$$

   (Note that the total is not quite 100% because of rounding.)

3. Which state or states is approximately "twice as toxic" as Alaska.

   Solution: Both Nevada and Utah are close to "twice as toxic" as Alaska.

F. Look at the plot in Figure 3-28 on page 71 of the text, which shows the gray wolf population in Wisconsin from 1980 to 2002.

1. Note that the portion of the graph representing the years 1980 to 1992 looks quite linear. Estimate a linear model for this period by using the line through the two points $(1980, 50)$ and $(1990, 60)$. (Determine the equation of the line through those points.)

   Solution: If we make the actual year be x and the wolf population y, then the equation of the line would be

$$y - 50 = \left(\frac{60 - 50}{1990 - 1980}\right)(x - 1980), \text{ or } y = x - 1930$$

   (It would also be possible to use $x =$ years since 1980, so $x = 0 \Leftrightarrow 1980$ and $x = 10 \Leftrightarrow 1990$. Doing it that way, the equation would be $y = x + 50$.)

2. Do the same for the period 1993-2002 using the two points $(1993, 70)$ and $(2002, 325)$.

   Solution: With $x =$ actual year and $y =$ wolf population, we have

$$y - 70 = \left(\frac{325 - 70}{2002 - 1993}\right)(x - 1993), \text{ or } y = 28.3x - 56331.9$$

3. What are the slopes of these two lines? What are the units of the slope and what do they represent in real world terms?

   Solution: The slope of the first line is 1 and the slope of the second is 28.3. These represent the growth rate of the wolf population per year in the two different time periods.

4. The graph in the spreadsheet (see course homepage) shows the least squares regression line for the whole data set, together with the computed slope and intercept values for the regression line. Which method seems superior in this case – using two different lines for different time periods, or one regression line for the whole time? Explain how you are making this decision.

   Solution: In this case it seems pretty clear that the two separate lines do a better job of approximating the actual wolf populations over the years in the two time periods. Using a single line for the whole period gives a consistent pattern in the residuals and is not a close fit.