MONT 108N – Mathematics Through Time
Problem Set 4 – Euclidean Geometry
due: November 5, 2010

I. Proposition I.5 in the *Elements* proves the equality of the base angles in an isosceles triangle using only Propositions I.1 - I.4. We indicated the main idea of the construction for the proof in class on 10/27. Work out a complete proof of this statement, and give a reason for each statement. Is your chain of reasoning exactly the same as Euclid's? (There are correct proofs that are slightly different, of course.)

II. In many modern treatments of plane geometry, an alternate form of the 5th Postulate is used, which looks like this: *Given an infinitely produced line ℓ and a point P not on ℓ, there is exactly one infinitely produced line through P that is parallel to ℓ.* This is certainly simpler than Euclid's Postulate 5, and it can be seen that they are equivalent.

A) Which Proposition in Euclid gives a construction for this parallel line?
B) Using this form of the 5th Postulate and Propositions I.15 and I.29 of the *Elements*, prove that the sum of the angles in any triangle is equal to a straight angle (that is, the sum is has angle measure $180°$; Euclid did not use the notion of a numerical measure for angles, though).
C) Deduce that the sum of the interior angles of a convex polygon with n sides is equal to $n - 2$ straight angles. (This is most easily done as a *proof by mathematical induction* to use the modern term. See me in office hours if you don't know what that means!)

III. A deductive sequence concerning areas giving an alternative to Euclid's derivation of the area formulas for parallelograms and triangles and another interesting fact. Start from the assumption that the area of a rectangle is the product of its dimensions (length times width). Using *only facts from Propositions I.1 - I.33* and previously proved parts of this problem, prove the following. (This means, for instance, that once you establish A, you can use that fact in the proof of B and similarly for the other parts.)

A) The area of a parallelogram is equal to the product of its base and altitude.
B) The area of a triangle is equal to the product of any side and the altitude on that side.
C) The area of a right triangle is equal to one half the product of its two legs.
D) The area of a triangle is equal to half the product of its perimeter and the radius of its inscribed circle. (For this part you will also need to assume the fact that a tangent line to a circle meets the radius at the intersection point in a right angle. This fact is proved in Euclid only in Proposition 18 of Book III.)