MONT 108N – Mathematics Through Time
Problem Set 3 – Babylonian Mathematics
due: Friday, October 8

I. A Old Babylonian tablet from about 1700 B.C.E (now held by the Louvre in Paris) has the following problem: "Find how long it will take a certain sum of money to double itself at compound annual interest of 20%." This means that you will have $1.2\times$ the original amount after 1 year, $(1.2)^2\times$ the original amount after 2 years, $(1.2)^3\times$ the original amount after 3 years, and so on. The question is: How many years will be needed until you have twice the original amount? (Fractional parts of years are also allowed.)

A) The Babylonian method of solution (written with base 10 numbers and in modern language, of course) was this: First compute the powers to see that $(1.2)^3 = 1.728$ and $(1.2)^4 = 2.0736$. So the doubling will happen between the 3rd and 4th year. To find the doubling time, find the point on the straight line through $(3, (1.2)^3) = (3, 1.728)$ and $(4, (1.2)^4) = (4, 2.0736)$ with $y = 2$. The x-coordinate of that point is the doubling time. Carry out the calculations to find this time.

B) The Babylonian tablet gives the answer by this method as the base 60 number

$$(3.47 : 13 : 20)_{60}$$

(with fractional part). Is this correct (does it agree with with you did in part A)?

C) Is this method exact or an approximation? Explain.

D) Solve the problem *exactly* by modern methods and compare with the Babylonian answer. How different are they? *Hint:* logarithms.

II. Old Babylonian tablets containing values of $n^3 + n^2$ for $n = 1, ..., 30$ has been found.

A) Make such a table for $n = 1, \ldots, 10$.

B) Use it to solve the cubic equation $x^3 + 2x^2 = 3136$. (Note: The trick is to multiply both sides of the equation by an appropriate number first, then consult the table!) *Answer: $x = 14$, but full details of how this is derived must be shown for credit.*

C) A tablet of about 1800 B.C.E. from Susa in present-day Iran asks for a solution of the system of equations

$$\begin{aligned} xyz + xy &= 7/6 \\ y &= 2x/3 \\ z &= 12x \end{aligned}$$

Use the last two equations to eliminate y, z and get an equation in x alone. Then use your table from part A) to find the solution.