MONT 108N – Mathematics Through Time
Problem Set 2 – Egyptian Mathematics
due: in class on September 27, 2010

I. Egyptian multplication and division

A) Multiply by successive doubling (that is, "the Egyptian way"): 434×127.
B) Multiply by successive doubling: 252×59.
C) Divide by the Egyptian method: $167 \div 18$.
D) What *advantage(s)* does the Egyptian method of multiplication have over the method commonly used now? What shortcomings?

II. Egyptian fractions – the table of conversions of $2/n$ into unit fractions from the Rhind papyrus is given in modern notation on page 44 in *The Babylonian Theorem.* Each row gives an expansion

$$\frac{2}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$$

for one of the odd integers $3 \leq n \leq 101$. (if there is no entry for c and/or d, then those terms are not present).

A) Verify that the rows for $n = 13$ and $n = 29$ are correct.
B) Use the table to find a decomposition of $\frac{4}{27}$ into unit fractions.
C) Show that for any positive integer n,

$$\frac{2}{n} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \frac{1}{6n}.$$

Yet this expansion appears only once in the table from the Rhind papyrus. For which n did the compiler of the table fall back on this form? Why might they have preferred to avoid using this?

III. *The* **seqt** *of a pyramid.* The Egyptians measured the steepness of a face of a pyramid by the ratio of the "run" (measured in hands) to the "rise" (measured in royal cubits). There were 7 hands in a royal cubit. See page 32 in *The Babylonian Theorem.* This ratio was called the *seqt* of the pyramid, measured in units of *hands per royal cubit.*

A) Problem 56 of the Rhind papyrus asks the solver to find the *seqt* of a pyramid 250 royal cubits tall, with a square base 360 royal cubits on a side. The answer is given as $5 + \frac{1}{25}$ hands per royal cubit. Is this correct?
B) The great pyramid at Giza (built by the Pharaoh Khufu, finished about 2560 B.C.E., and the only one surviving of the "seven wonders of the ancient world") has a base 440 royal cubits on a side and a height of 280 royal cubits. What is its *seqt*?

IV. *An Egyptian approximation to π.* We take π to be the ratio between the area of a circle and the square of the radius: $\pi = \frac{A}{r^2}$.

A) In the Rhind papyrus, the area of a circle with diameter d is given as

$$A_{circ} \doteq \left(\frac{8}{9}d\right)^2.$$

(This is not exactly correct, but yields decent approximate results.) What is the implied approximation to the value of π here?

B) Here's one way this may have been derived. Take a square of side 9 units, divide the edges into three equal pieces, and cut off four corner triangles with side length 3 to make an octagon. The octagon is pretty close in area to the circle inscribed in the original square. Explain why the octagon has area $63 \doteq 8^2$, and then explain how the approximation $A_{circ} \doteq \left(\frac{8}{9}d\right)^2$ would result.

Essay

Write about 1 page on your reactions to one of the following:

1) The talk by Tom Zetterstrom
2) The photography exhibit of Tom Zetterstrom's work in the Cantor gallery
3) The talk by John Hess on Darwin's trip to the Galapagos Islands

In particular, did your experience of the talk or show affect how you view the natural world and our place in it? If so, how? If not, why not? Be sure to state carefully who the "we" is that the "our" refers to for you here – humans generally, Americans, Holy Cross students, some other group.

Please send your essays as Word .doc files to `jlittle.holycross.edu` separately from your problem solutions