MONT 108N – Mathematics Through Time
Problem Set 1
due: in class on September 20, 2010

I. "Refresher" on arithmetic with fractions (rational numbers) and radicals. Calculate:

A) $\frac{3}{7} + \frac{4}{9}$

B) $\frac{4}{19} \times \frac{5}{12}$ (and simplify)

C) $\frac{5}{6} \div \frac{3}{4}$ (and simplify)

D) $\frac{1}{4} + \frac{\frac{1}{4}}{1-\frac{1}{3}}$ (and simplify)

E) Simplify: $\frac{\sqrt{200}}{\sqrt{8}}$.

F) Write the number $\frac{1}{\sqrt{3}-1}$ in the form $a+b\sqrt{3}$ with a, b rational numbers by multiplying by the same number in the numerator and denominator:

$$\frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

and simplifying.

II. For us (and all mathematicians), an integer is *even* if it equals 2 time another integer – so the even numbers are

$$\ldots, -8 = 2 \times (-4), -6 = 2 \times (-3), -4, -2, 0 = 2 \times 0, 2, 4, 6, 8, \ldots.$$

All the other integers are *odd*.

A) Show that if an integer n is even, then n^2 is also even.

B) Show that if an integer n is odd, then n^2 is also odd. (Hint: The odd numbers all have 1's digit equal to 1,3,5,7, or 9.)

C) Show that if n is an integer and n^2 is even, then n is also even. (This is the "converse" of the statement in part A. You may use part B here if it helps!)

III.

A) Express the following base 10 numbers using base 2 and base 8: 12, 46, 100.

B) Express the following base 2 numbers in base 10: $(10011)_2$, $(111111)_2$.

C) Express the following base 8 numbers in base 10: $(4006)_8$, $(123)_8$.

D) Suppose we wanted to use a base 60 positional number system. How many base 60 digits would we need? How would you write the base 60 digits as base 10 numbers?

E) Short answer: Why is having the 0 digit (almost) a necessity for a positional number system with base b (b = any positive integer)? Explain.

IV. Solve by factoring (you may need to experiment a bit for the later ones!)

A) $x^2 - 5x + 6 = 0$

B) $x^2 + 4x + 4 = 0$

C) $x^2 + 12x - 108 = 0$

D) $3x^2 + 28x + 32 = 0$

E) $t^6 - 6t^3 + 9 = 0$

V. Solve using the quadratic formula

A) $x^2 + x - 1 = 0$

B) $x^2 + 8x = 3$

C) $2x^2 + 7x + 3 = 0$