

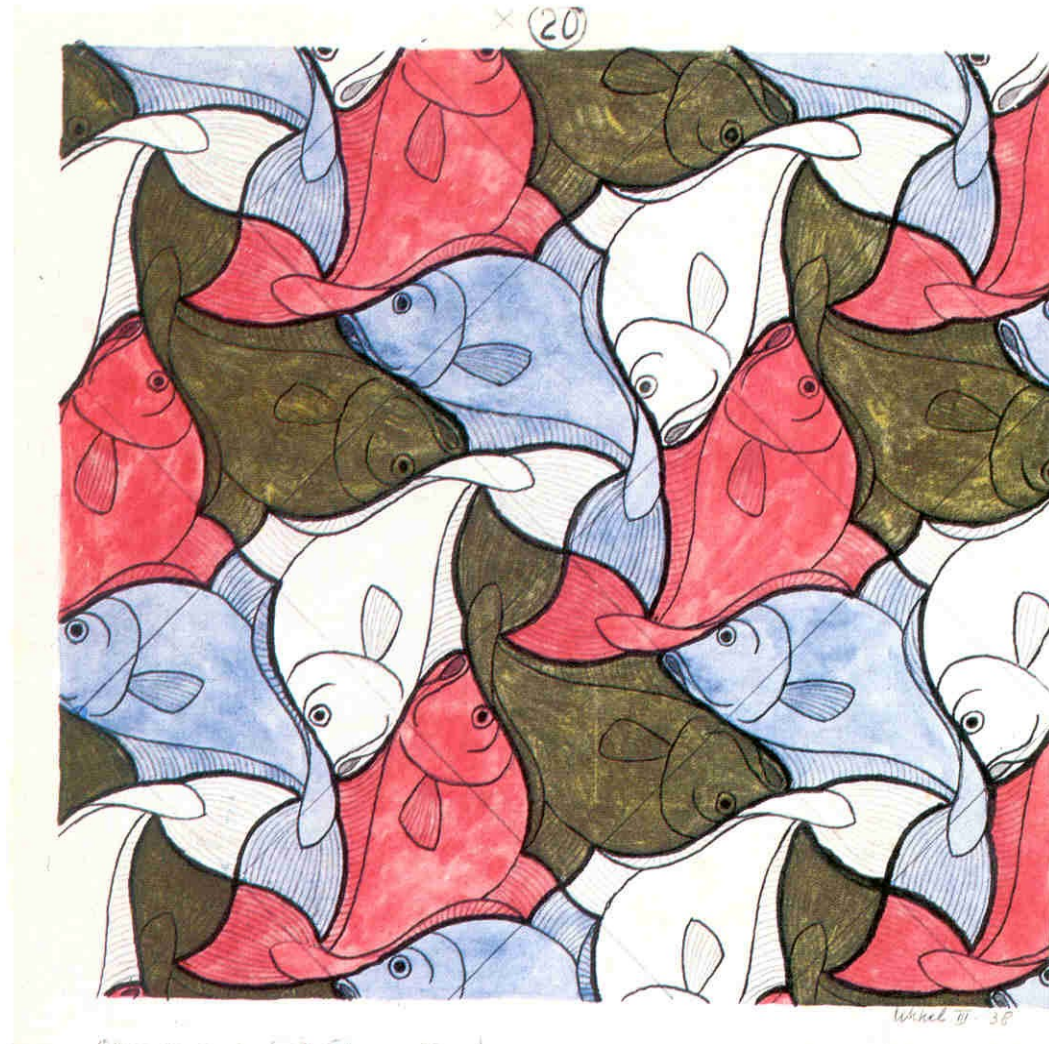
# MONT 109N – Symmetry in music

- Plan for today:
- Recap of ideas of *symmetry*, see how they might apply in *music*.
- Examples from J. S. Bach, *A Musical Offering*
- Discussion – was Bach “doing mathematics?” If not, what was he doing?

# Mathematical symmetry

- The precise mathematical understanding of symmetry: *invariance under a transformation*.
- For instance, bilateral symmetry is invariance under reflection across a plane in three dimensional space (or across a line in the plane).
- Other types of transformations can be considered and other symmetries studied.
- Geometrical examples: translations, rotations. Each gives a corresponding form of symmetry.

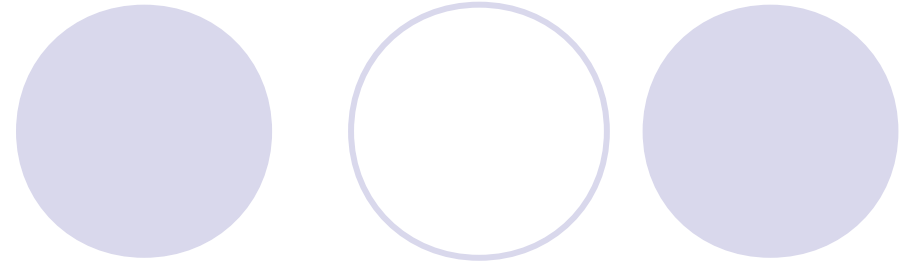
# Visual symmetry – M.C. Escher



# How does this apply to music?

- The idea is that various “dimensions” of a musical composition
  - the points in time when different events occur
  - duration and spacing of events
  - the pitches that sound
  - dynamics, instrumentation, expression directions, ...
- can also be subjected to various types of *transformations* such as augmentation or diminution (*stretching out or speeding up time*), retrograde motion (*time reflection*), transposition or inversion in pitch, ...

# Symmetry in Music



- These transformations can be used by composers to build up pieces locally and even globally.
- A musical composition can even exhibit the kind of invariance (or “idealized” invariance) that we see in patterns such as the Escher drawing.



# An extended example

- We have been discussing possible transformations of music and symmetry properties from a more or less “theoretical” point of view so far.
- The next goal is to explore an extended series of examples to see some of these ideas “in action”.
- Perhaps the best single example -- *A Musical Offering*, by Johann Sebastian Bach (1685-1750).

# Background on Bach



- Lived 1685 – 1750 C.E.
- Worked at times as a composer/performer of music in courts of small principalities in present-day Germany.
- Most important position – “Cantor” at St. Thomas in Leipzig (a Lutheran church)
- At Leipzig – he was organist, choirmaster, wrote a tremendous amount of music for church services, also teacher in the St. Thomas choir school.

# More on Bach



- Best known as a keyboard (organ, harpsichord, etc.) performer and improviser during his lifetime.
- Worked in a style that was becoming *old-fashioned* even during his lifetime
- Baroque period roughly 1600 – 1750.
- Bach was the “last gasp” and summing up of this tradition.



# Important aspects of Bach's music

- *Polyphony* (the interplay of several musical voices sounding together) and *counterpoint* (musical techniques for doing that) key preoccupations.
- Especially toward the end of his life, he tended to concentrate on the “strictest” contrapuntal forms – *canon* and *fugue*
- “*Pure music*” – often limited directions given for how it was to be performed

# History of “The Musical Offering”

- May 7, 1747 – J.S. Bach visits the palace of Frederick II of Prussia (“Frederick the Great”) at Potsdam (outside Berlin).
- Frederick had several new *pianofortes* made by Gottfried Silbermann; recall Bach was known at the time primarily as a performer.
- Several of his sons (including Carl Phillip Emmanuel, who was employed by Frederick) were better known composers.

# History, continued



- Frederick was himself an accomplished flutist and composer of flute music (all in very up-to-date “*galant*,” “*rococo*” style);
- But apparently he was also a connoisseur of older music and asked Bach to *improvise a fugue* in 3 voices on a theme he provided.
- After Bach succeeded there, was also asked to do the same in 6 voices(!) He “begged off” on that one!

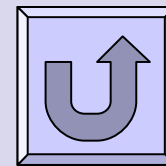


# The canon at the unison

- What symmetry is involved here?

The image displays two systems of musical notation for Violino I and Violino II. The first system is marked with a '2.' and shows the beginning of the canon. Violino I starts with a melodic line, and Violino II enters with the same line after a short interval. The second system continues the canon, showing the two violins playing the same melody in unison. The notation includes treble and bass clefs, a key signature of two flats, and a common time signature. Trills are indicated with 'tr' above certain notes. The score is presented in a clear, black-and-white format.

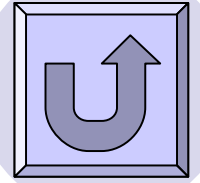
# A two-voice canon



Canon a 2. J. P. Kirnberger

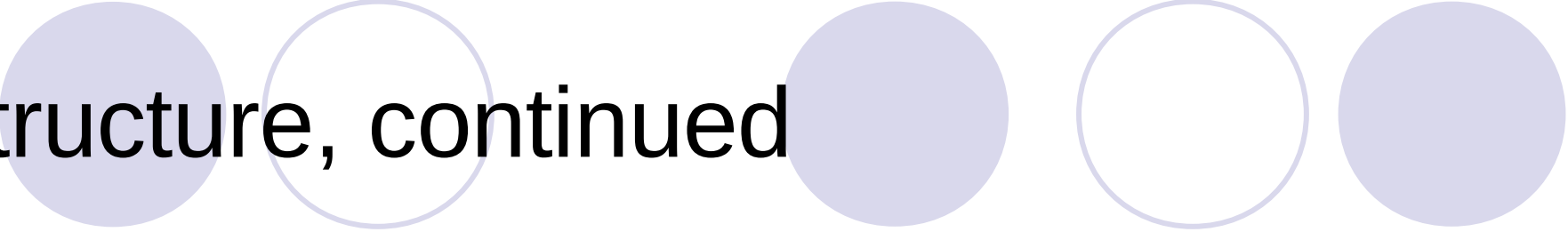
1.

# Structure of this canon



- The piece is 18 measures long (played twice on recording).
- In measures 1 – 9, the top voice plays the Royal Theme, while the bottom voice plays an accompanying figure, with faster motion.
- In measures 10 – 18, the top voice switches to the accompanying figure, while the lower voice plays the Royal Theme, *but both are moving backwards in time.*
- *Also called a canon cancrizans = “crab canon”*

# Structure, continued



- In other words, this canon is a musical “palindrome” -- as a whole piece of music, it is symmetric under reflection in time around the barline between measures 9 and 10. It would sound the same played forwards or backward in time(!)
- Another point: Who's Kirnberger? We didn't mention this before, but one curious feature of Bach's *Musical Offering* was that the canon sections were actually left as **puzzles** for other musicians to decipher. Kirnberger was the first solver!



# Bach's actual *puzzle canon* notation

- <http://mathcs.holycross.edu/~little/Montserrat1011/MusOffCanon1.pdf>

# The canon by contrary motion

- What symmetry transformation is involved here?

a 2. *Per motum contrarium.* J. P. Kirnberger

3.

The image shows a musical score for a canon by contrary motion. The score is written in three systems, each with three staves (treble, alto, and bass clefs). The key signature has two flats (B-flat and E-flat), and the time signature is common time (C). The first system is marked '3.' and the second system is marked 'a 2. Per motum contrarium.' The music shows a canon where the second voice enters in contrary motion to the first. The third system continues the piece with various rhythmic patterns and rests.

# The modulating canon

- “As the notes rise, so may the glory of the King” -- was Bach being ironic?

a 2. (Per tonos.)\*

J. P. Kirnberger

5.

Finis



# Some food for thought

- Early examples of canons in medieval music definitely have an element of religious symbolism -- the first voice sets out The Law in musical terms and the other voices follow obediently.
- What would you expect music written to flatter a royal patron and extol his “greatness” to sound like? Do these pieces sound like that?

# Questions for further thought



- Not all musicians use these ideas, and even those who do don't *always* use them. What might be a reason for incorporating these ideas in a piece?
- When they do, are they “doing mathematics”?
- “Music is the hidden arithmetical exercise of a mind unconscious that it is calculating” – Gottfried Leibniz.



# A final thought

Is this sort of “play” with symmetry transformations *limited* to art, mathematics, and music? Are we perhaps saying something about the way human intelligence works in general?