MONT 109N – Mathematics Across Cultures
Discussion/Problem Set 5 – Indian Mathematics
April 8, 2011

Background and Assignment

In this assignment, we want to look at a sample of Indian mathematics from the "classical" period (roughly 400 to 1200 C.E.). Group writeups are due in class on Friday, April 15.

Questions

A. The following problems appear in a text by the mathematician Mahavira (circa 850 C.E.). Solve them using our modern notation and methods. (The colorful directions are a semi-literal translation of the original :))

1. Of a collection of mangoes, the king took 1/6, the queen 1/5 of the remainder, the chief princes $1/4, 1/3, 1/2$ of the successive remainders, and the youngest child took the remaining 3 mangoes. O you who are clever in miscellaneous problems on fractions, give the measure of that collection of mangoes.

2. The total price of 9 citrons and 7 fragrant wood apples is 107. Again, the total price of 7 citrons and 9 fragrant wood apples is 101. O you arithmetician, tell me quickly the price of a citron and a wood apple here, having distinctly separated those prices well.

3. One fourth of a herd of camels was seen in the forest; twice the square root of that herd had gone to the mountain slopes; and three times five camels remained on the river bank. O smiling maiden with the shining eyes, what is the numerical measure of that herd of camels?

B. Bhaskaracharya (circa 1150 C.E.) developed the following identities for the square roots of numbers of the form $a \pm \sqrt{b}$ (a,b integers or rational numbers more generally):

$$\sqrt{a \pm \sqrt{b}} = \sqrt{(a + \sqrt{a^2 - b})/2} \pm \sqrt{(a - \sqrt{a^2 - b})/2}.$$

1. Show that the formula is valid (square both sides and simplify).

2. Use the formula above to express $\sqrt{17 + \sqrt{240}}$ as a sum of two square roots.

C. Probably the high point of Indian geometry is a formula for the area of a *cyclic quadrilateral* found by Brahmagupta (circa 630 C.E.). A convex quadrilateral $PQRS$ is said to be *cyclic* if all four vertices P, Q, R, S lie on one circle (this is not automatic for sets of four points in the plane). Then Brahmagupta's formula gives a way to compute the *area* A of the quadrilateral if the lengths of the sides are known. Say the four sides of the cyclic quadrilateral are a, b, c, d in some order. Also, let $s = \frac{a+b+c+d}{2}$ be the "semiperimeter." Then the area A is given by

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}. \qquad (1)$$

(Some of you may know a very similar formula for triangles called *Heron's formula*. Heron was an Alexandrian mathematician and engineer who lived roughly 10 - 70 C.E. The result known as Heron's formula says that the area of a *triangle* with sides a, b, c and semiperimeter s is

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

It is thought that Brahmagupta was probably familiar with that result. And indeed, it is possible to see Heron's formula *as a limiting case of Brahmagupta's* if you let two of the vertices of the cyclic quadrilateral coincide, so $d \to 0$, for instance(!))

1. Show that if a quadrilateral has perpendicular diagonals then the sums of the squares of the lengths of opposite pairs of sides are equal. That is, if the two pairs of opposite sides are a, c and b, d, then $a^2 + c^2 = b^2 + d^2$.

2. Conversely, show that if $a^2 + c^2 = b^2 + d^2$, then the diagonals of the quadrilateral are perpendicular.

3. Brahmagupta showed that if $\alpha^2 + \beta^2 = \gamma^2$ and $A^2 + B^2 = C^2$ are two Pythagorean triples, then a quadrilateral having the lengths $\alpha C, \gamma B, \beta C, \gamma A$ for consecutive sides has perpendicular diagonals. Show this.

4. Suppose $(\alpha, \beta, \gamma) = (3, 4, 5)$ and $(A, B, C) = (5, 12, 13)$. Find the sides, diagonals, and the area of the quadrilateral formed as in part 3. (Hint: You can use equation (1) for the area; do you see why?)